Sequences.05.11.2013

Sequences.

1. Let a_n be a geometric sequence, defined by $a_{n+1} = q \cdot a_n$, where $q \neq 0$.

1. What is the limit of a_n (consider the case $q > 1$ and $q < 1$ separately)
2. What is the formula for a_n (use the induction to prove it)
3. Show that $S_n = a_1 + a_2 + \cdots + a_n = a_1 \cdot \frac{q^n - 1}{q-1}$. Use the induction.

2.
Find the limit of a sequence a_n, where

1. $a_n = \frac{(0{,}5)^n}{n+1}$
2. $a_n = \sqrt{3n^2 + 4n - 6} - 2n$
3. $a_n = \frac{2^{2n+1} - 7}{2^n + 4}$
4. $a_n = (4^n + 5^n + 6^n)^{\frac{1}{n}}$
   Hint: use the sandwich theorem (squeeze theorem, 3 sequences theorem)
5. $a_n = (1 + \frac{4}{n})^n$. What is the relation of the sequence a_n with the compound interest formula?
   Hint: Use the fact that the number e is a limit of some special sequence
6. $a_n = (\frac{n}{n+1})^n$