

## Classes 1

1.

1. Let  $A, B, C$  be variables with values in the set of students. Every student learns at least two languages from the set  $\{L_1, L_2, L_3, L_4\}$ . Is the following sentence always true? If  $A$  and  $B$  learn at least one language together and  $B$  and  $C$  learn at least one language together then  $A$  and  $C$  learn at least one language together.

2. Let  $a, b, c$  be natural numbers. Is the following sentence true?

If  $b|a$  and  $c|a$  then  $(bc)|a$ .

*Remark: symbol  $b|a$  means that  $a$  is divisible by  $b$ . In other words,  $a = b \cdot c$  for some natural number  $c$ .*

3. Is the following sentence a tautology:

1.  $[(p \vee q) \wedge \neg p] \Rightarrow q$

2.  $[(p \Rightarrow q) \wedge (q \Rightarrow p)] \Rightarrow (p \vee q)$

4. Consider the formula  $\Phi(m, x)$  with two real-valued variables  $m, x$ :

$$\Phi(m, x) \Leftrightarrow mx^2 + mx + 4 > 0.$$

Which of the sentences are true:

a)  $\forall m \forall x \Phi(m, x)$ .

b)  $\forall m \exists x \Phi(m, x)$ .

c)  $\exists m \forall x \Phi(m, x)$ .

d)  $\exists m \exists x \Phi(m, x)$ .

e)  $\forall x \exists m \Phi(m, x)$ .

f)  $\exists x \forall m \Phi(m, x)$ .

5. On the real plan, draw the set of points with coordinates  $(x, y)$  given by the following condition: a)  $x^2 + y^2 \geq 2$ ;

b)  $|x| + |y| \leq 2$ .

6. Let

$$A = \{(x, y) : y < x\}, \quad B = \{(x, y) : \frac{y}{x} < 1\}.$$

Find and draw the following sets:

$$A \cap B, \quad A \cup B, \quad A \setminus B, \quad B \setminus A.$$

7. Let

$$A = \{(x, y) : x^2 + y^2 \leq 2\}, \quad B = \{(x, y) : x + y > 0\}.$$

Draw the following sets on the real plane:

$$A \cap B, \quad A \cup B, \quad A \setminus B, \quad B \setminus A.$$

8. Using the mathematical induction, show that for any natural number  $n$ :

1.  $(1 + a)^n \geq 1 + na$ , where  $a > -1$

2.  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$