Topic XI

The derivative of a function

1. Calculate the derivative of function f wherever it exists.

a) $f(x) = \frac{1}{x^3}$. b) $f(x) = \frac{1}{\sin x}$. c) $f(x) = \frac{x+1}{x-1}$.

d) $f(x) = \sin^3 x$. e) $f(x) = \sqrt[3]{x}$. f) $f(x) = \sqrt[3]{1+x^3}$.

g) $f(x) = e^{-x}$. h) $f(x) = e^{x^2}$. i) $f(x) = x \ln x$.

j) $f(x) = \log_2 x$. k) $f(x) = \log_x 2$. l) $f(x) = x^x$.

m) $f(x) = x^{x^2}$. n) $f(x) = (x^x)^2$.

2. Examine if the following function is differentiable at point $x_0 = 0$.

a) $f(x) = x|x|$;

b) $f(x) = |x|^3$;

c) $f(x) = |\sin^3(x)|$;

d) $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{dla} \quad x \neq 0, \\ 0 & \text{dla} \quad x = 0; \end{cases}$

e) $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{dla} \quad x \neq 0, \\ 0 & \text{dla} \quad x = 0; \end{cases}$

f) $f(x) = \begin{cases} e^{-\frac{1}{x}} & \text{dla} \quad x > 0, \\ 0 & \text{dla} \quad x \leq 0. \end{cases}$

3. Find the equation of the line which is tangent to the graph of function $f(x) = x^x$ at point $(2, 4)$.

4. Calculate the angles under which the graphs of functions $f(x) = x^2$ and $g(x) = x^3$ intersect themselfs.

5. Calculate the angles under which the graphs of functions $f(x) = \sqrt[3]{x}$ i $g(x) = x^3$ intersect each other.

6. Using the theorem about the derivative of an inverse function calculate the $(f^{-1})'(0)$, where $f(x) = x + \sin x$.

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable odd function(ie. $f(-x) = -f(x)$ for any x). Show that $f'(x)$ is an even function.

8. Let $f : \mathbb{R} \to \mathbb{R}$ be an even function, differentiable at point $x_0 = 0$. Show that $f'(0) = 0$.

9.Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Is this true that for any $x_0 \in \mathbb{R}$ there exists a pair of points $a < x_0 < b$ which is such that $f'(x_0) = \frac{f(b)-f(a)}{b-a}$?

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