Temat XIII.

l'Hospital's rule, elasticity and Taylor's formula.

1. Using L'Hospital's rule (if necessary), calculate the following limits:

a) $\lim_{x\to 0} \frac{e^x - 1}{\sin 2x}$; b) $\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}$; c) $\lim_{x\to 0} \frac{\sin x}{x + x^2}$;

d) $\lim_{x\to +\infty} \frac{e^x}{x + x^2}$; e) $\lim_{x\to +\infty} \frac{\ln x}{\sqrt{x}}$; f) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$;

g) $\lim_{x\to 0} \left(\frac{1}{x} - \operatorname{ctg} x\right)$; h) $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{1}{\ln(x-1)}\right)$; i) $\lim_{x\to +\infty} \left(\sqrt{x^2 + 3x} - x\right)$;

j) $\lim_{x\to 0^+} x \ln x$; k) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$; l) $\lim_{x\to +\infty} x \left[\left(1 + \frac{1}{x}\right)^x - e)\right]$.

2. Elasticity. *Trzy zadania z zestawu doc. M. Krycha.*

a)Let D be a demand function varying inversely proportional to the price p. If the p increases by one percent, what is the percentage change in demand.

b)Assume that, as a function of the price p, the demand for chocolate from the the rural population is three times smaller than the demand from the urban population with elasticity -0.8 in the rural case, and elasticity 0.3 in the urban case. Approximately what percentage change in global demand for chocolate will result from an increase in the price by 1 %?

c) Let $C(q)$ denote the cost of producing q widgets, and $c(q) = \frac{C(q)}{q}$ the average cost of producing one widget. Assume that C extends to a differentiable function of $(0, \infty)$ and show that $C'(q) = c(q)(1 + E_q(c))$.

3. Calculate the Taylor series together with its radius of convergence for the following functions:

a) $f(x) = \frac{1}{1+x}$, $x_0 = 0$; b) $f(x) = \frac{1}{2+x^2}$, $x_0 = 0$;

c) $f(x) = \frac{1+x}{2+3x^2}$, $x_0 = 0$; d) $f(x) = \frac{1}{(x-2)(4-x)}$, $x_0 = 3$;

e) $f(x) = \frac{1}{(x-2)(4-x)}$, $x_0 = \frac{5}{2}$; f) $f(x) = \frac{1}{(x-4)^2}$, $x_0 = 5$;

g) $f(x) = (1+x)^2$, $x_0 = 2$; h) $f(x) = (1+x)^{\frac{1}{2}}$, $x_0 = 0$

i) $f(x) = (1-x)^{\frac{1}{3}}$, $x_0 = 0$; j) $f(x) = (1-x)^{-\frac{1}{3}}$, $x_0 = 0$;

k) $f(x) = \sin(x^2)$, $x_0 = 0$; l) $f(x) = \sin^2(x)$, $x_0 = 0$.

4. Use the Taylor formula together with the Lagrange formula for the error to calculate:

a) The number e with 5 decimal places of accuracy.

b) The number $\sqrt{105}$ with 3 decimal places of accuracy.

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