Temat VI

Ciągi II.

1. For each of the following sequences, calculate the limit if the limit exists. If the limit does not exist, explain why it does not exist.

a) $a_n = \frac{n^2+(-1)^n\cdot n}{(n+\sqrt{3})^2}$,

ą) $a_n = \frac{\left(\sqrt{n+8\sqrt{n}}\right)^4}{\left(n+3\sqrt{n}\right)^2}$,

b) $a_n = \frac{n(\sqrt{3n}-7)^2}{(\sqrt{n+2}+2)^4}$,

c) $a_n = \sqrt{\frac{n+(-1)^n\sqrt[4]{n}}{(3n+\sqrt{2n})^2}}$,

ć) $a_n = \frac{\binom{n+2}{n+1}^2}{2+4+6+\cdots+2n}$,

d) $a_n = \sqrt{n+1} - \sqrt{n}$,

e) $a_n = \sqrt{n^2+5n} - \sqrt{n^2-n}$,

ę) $a_n = \sqrt{5n^2-3n} - n\sqrt{5} + 8$,

f) $a_n = \sqrt{n^4+3n^2+5} - \sqrt{n^4-n^2+n}$,

g) $a_n = \frac{\sqrt{n^2+\sqrt{n+1}}-\sqrt{n^2-\sqrt{n-1}}}{\sqrt{n+1}-\sqrt{n}}$,

h) $a_n = \frac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt{n^2+3}}$,

i) $a_n = \frac{1^2}{n^3} + \frac{3^2}{n^3} + \frac{5^2}{n^3} + \cdots + \frac{(2n-1)^2}{n^3}$,

j) $a_n = n(\sqrt[3]{n^3+n+2} - n)$,

j *bis*) $a_n = \sqrt[3]{n(n+1)^2} - \sqrt[3]{n(n-1)^2}$,

k) $a_n = \frac{(3n)!}{(n!)^3}$,

l) $a_n = \frac{\cos(n!)}{\sqrt{n}}$,

ł) $a_n = \frac{\sqrt[n]{3}-1}{2+(-1)^n}$,

m) $a_n = \frac{\sqrt[n]{n}-1}{3+(-1)^n}$,

n) $a_n = \sqrt[n]{1+\sqrt{\binom{n}{2}}}$,

ń) $a_n = \sqrt[n]{2^n+n^2}$,

o) $a_n = \sqrt[n]{3^n+5^n+7^n}$,

ó) $a_n = \sqrt[n]{\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n + \left(\frac{1}{4}\right)^n}$,

p) $a_n = n\left(\frac{1}{n^2+1} + \frac{1}{n^2+2} + \cdots + \frac{1}{n^2+n}\right)$,

q) $a_n = \frac{1}{n^2+1} + \frac{2}{n^2+2} + \cdots + \frac{n}{n^2+n}$,

r) $a_n = \frac{1^4+2^4+3^4+\cdots+n^4}{1^4+2^4+3^4+\cdots+n^4+(n+1)^4}$,

s) $a_n = \frac{1!+3!+5!+\cdots+(2n-1)!}{2!+4!+6!+\cdots+(2n)!}$, ś) $a_n = \left(1 - \frac{1}{n}\right)^n$,

t) $a_n = \left(1 - \frac{1}{n^2}\right)^{2n-1}$,

u) $a_n = \left(1 + \frac{1}{2^n}\right)^{2^{n+1}}$,

v) $a_n = \left(\frac{n-1}{n+3}\right)^{2n+1}$,

w) $a_n = \left(1 + \frac{(-1)^n}{n}\right)^{(-1)^n n}$,

x) $a_n = \left(\frac{n^2-1}{n^2}\right)^{2n^2-n}$,

y) $a_n = \left(\frac{1+\sqrt[n]{2}}{2}\right)^n$,

z) $a_n = \frac{F_{n+1}}{F_n}$, where F_n is the n-th Fibonacci number,

ź) $a_1 = 1, \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{5}{a_n}\right)$,

ż) $a_1 = 1, \quad a_{n+1} = 1 + \frac{1}{a_n}$.

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