A trip through XVAs

Fabio Marelli

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The views and opinions expressed in this article are solely my own and do not necessarily reflect the views and opinions of my current employer.

CVA: definition and formulas

Definition . . .

Credit Valuation Adjustment (**CVA**) is an adjustment to the risk-free price of an OTC derivative to account for the risk that the counterparty may default and hence the bank may not receive the full market value of the derivative.

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Thus CVA is commonly viewed as the **price** of counterparty risk.

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3. if the counterparty defaults at an unknown future time $\tau \Rightarrow$ introduce counterparty's default probabilites:

$$\mathrm{V}_R(t_0) = \mathrm{V}_{RF}(t_0) - (1 - RR_C) \cdot \sum_{i=1}^{N} \mathrm{DP}_C(t_{i-1}, t_i) \cdot \mathrm{D}(t_0, t_i) \cdot \max\left(\mathrm{V}_{RF}(t_i), 0\right)$$

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In general,

$$\mathrm{CVA}(t_0) = \mathbf{E}_{t_0}^{Q}\left[(1 - \mathrm{RR}_C) \cdot \mathrm{I}_{\{\tau_C < T\} \cup \{\tau_B \geq \tau_C\}} \cdot \mathrm{D}(t_0, \tau_C) \cdot \max\left(\mathrm{V}_{RF}(\tau_C), 0\right)\right]$$

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Similarly, *Debit Valuation Adjustment* (**DVA**) is an adjustment to the risk-free price of an OTC derivative to account for the risk that the bank may default and hence the counterparty may not receive the full market value of the derivative.

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The risky price of a derivative is given by

$$V_R(t_0) = V_{RF}(t_0) - \mathrm{CVA}(t_0) + \mathrm{DVA}(t_0).$$

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- Capital requirements perspective:
  1. CVA reduces the bank's CET. Therefore, RWA needs to decrease to meet the regulatory requirements;
  2. DVA is explicitly ruled out: "*This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital* [...]." (Basel III).

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- Independence: should we model the interdependence between the random variables?

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- *Variation margin*: it protects the bank from the current exposure that has already been incurred from changes in the mark-to-market value of the contract.
- *Initial margin*: it protects the bank from the potential future exposure that could arise from future changes in the mark-to-market value of the contract *during the time it takes to close out and replace the position in the event that the counterparty defaults*.

Counterparty risk mitigants - 2

- Netting agreements - for example, ISDA Master Agreement.
  1. If there is no netting: $\mathrm{V}^{+}(T) = \sum_i \max(V_i(T), 0)$.
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- Break clauses: allow early termination of a contract. They are *mandatory* or *optional*.

How collateral mitigates exposures

Let's assume that the derivative price V is always positive.

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   where CB is the *Collateral Balance* and MTA is the *Minimum Transfer Amount*.

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3. Update the Collateral Balance: $\mathrm{CB}(t_k) = \mathrm{CB}(t_{k-1}) + \mathrm{MCA}(t_k)$
4. Compute the *Collateralized mark-to-market*:

$$\mathrm{VColl}(t_k) = \mathrm{V}(t_k) - \mathrm{CB}(t_k)$$

Wrong Way Risk

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For example:

- a bank enters an oil swap with an airline: the bank pays a fixed rate, whereas the airline pays the oil price (floating).
- If the oil price increases, the bank will increase its exposure to the airline company. . .
- . . . and, at the same time, the airline company will have higher costs to buy fuel and, therefore, its default probabilities are likely to increase.

Wrong Way Risk: a toy example

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- We discretize the interval $[t_0, T]$ into N steps $t_1, \ldots, t_N = T$.

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- Define $Z_3 = \rho Z_1 + \sqrt{1-\rho^2}Z_2$, where Z_1 and Z_2 are independent $N(0,1)$;

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- Use Z_1 to generate a random step of the CIR model
- Use Z_3 to generate a default event:
  - generate a uniform number $u \in (0,1)$ by inversion $u = \Phi^{-1}(z_3)$
  - if $u \geq 1 - \mathrm{DP}_C(t_{i-1}, t_i)$, then a default occurs: $\tau = t_i$

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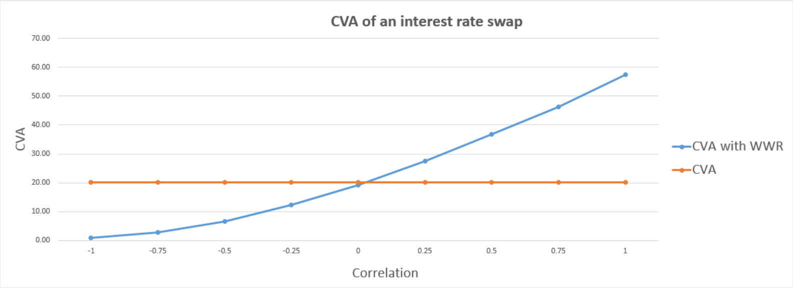

Figure: CVA: impact of correlation between interest rates and default on an interest rate swap

FVA – How funding costs arise

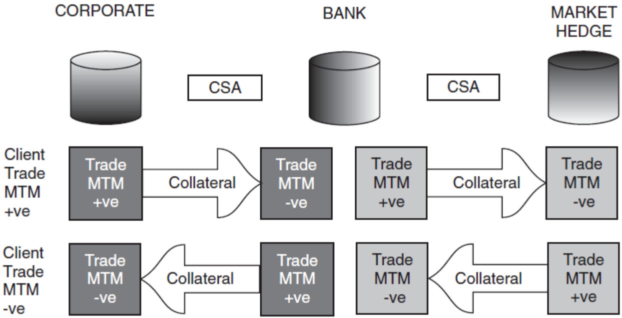

Figure: The case without no funding costs

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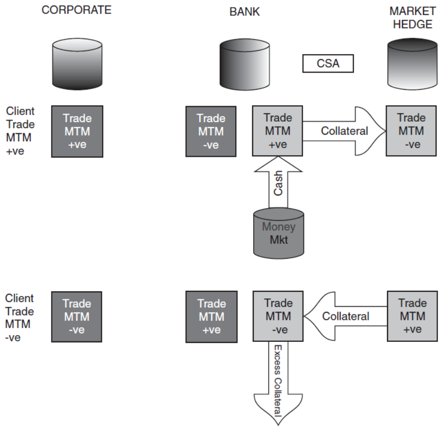

Figure: The case with funding costs

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$\Rightarrow$ Funding Valuation Adjustment (**FVA**) can be seen as the cost (or the benefit) for the bank to trade with an uncollateralized counterparty.

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⇒ Funding Valuation Adjustment (**FVA**) can be seen as the cost (or the benefit) for the bank to trade with an uncollateralized counterparty.

$$
\begin{aligned}
FVA = & -\sum_{i=1}^{N} \mathrm{SP}_B(0,t_i)\cdot \mathrm{SP}_C(0,t_i)\cdot \mathrm{FS}_B(t_{i-1},t_i)\cdot (t_i - t_{i-1})\cdot V_{RF}^{+}(t_i) \\
& -\sum_{i=1}^{N} \mathrm{SP}_B(0,t_i)\cdot \mathrm{SP}_C(0,t_i)\cdot \mathrm{FS}_L(t_{i-1},t_i)\cdot (t_i - t_{i-1})\cdot V_{RF}^{-}(t_i)
\end{aligned}
$$

An overview of the derivative market

The majority of collateral held by banks in the US is very liquid:

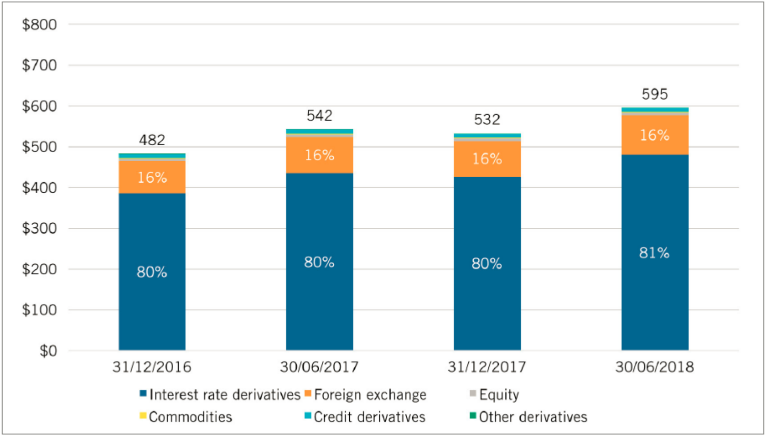
- 60.6% is held in cash (both U.S. dollar and non-dollar);
- 12.2% is held in U.S. Treasuries and government agency securities.

	Cash U.S. Dollar	Cash Other Currencies	U.S. Treasury Securities	U.S. Gov't Agency	Corp Bonds	Equity Securities	All Other Collateral
2018 Q3	37.5%	23.1%	10.1%	2.1%	2.0%	8.4%	16.8%
2018 Q2	38.3%	24.8%	9.9%	1.9%	1.9%	7.3%	15.9%
2018 Q1	37.7%	25.4%	10.5%	1.8%	2.1%	8.5%	14.0%
2017 Q4	37.6%	25.5%	10.3%	1.9%	2.5%	5.7%	16.4%
2016 Q4	40.1%	31.5%	8.1%	1.7%	1.6%	5.0%	12.0%
2015 Q4	43.7%	31.7%	4.6%	1.6%	1.4%	5.3%	11.7%

	Year-end 2017 (US$ billions)
IM posted	81.7
IM received	130.6
VM posted	631.7
VM received	893.7
Total margin exchanged	1737.6

Source: ISDA Margin Survey

Figure: Amount of VM and IM exchanged by Phase-one firms for their non-cleared derivatives at year-end 2017.



Source: BIS OTC Derivatives Statistics

Figure: Global OTC derivatives notional outstanding in US $ trillions.

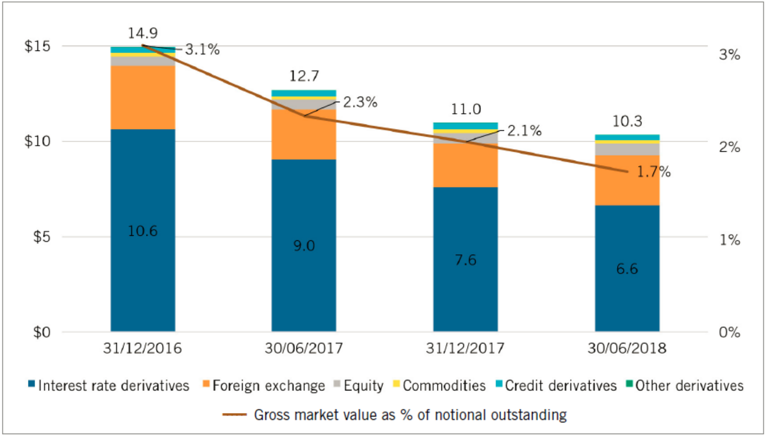


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Figure: Gross market value of global OTC derivatives in US $ trillions.

References

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Thank you!