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Counting lines on quartic surfaces

The fact that a compact complex 2-dimensional manifold X_d given as the set of zeroes of a degree-d homogenous polynomial in 3-dimensional projective space contains at most d(11d-24) lines was shown already by G. Salmon around 1840. Salmon, Cayley and Clebsch were also able to prove that the above claim is optimal for every cubic surface (i.e. for d=3).

It is a far more difficult question whether the above bound is sharp (resp. what is the maximum number of lines on X_d's) once we fix a degree d that exceeds 4. After a brilliant (but based on false claims on configurations of lines) argument by B. Segre (1943), the first correct proof of the claim that a smooth quartic surface contains at most 64 lines was given in 2012 (M. Schuett-S. R.).

Hardly anything is known on number and configurations of lines on degree-d surfaces when d is at least 5.

In my talk I will discuss the classical argument by Segre and sketch the proof of the sharp bound for d=4. I will explain why some claims by Segre are false by providing a detailed picture of the geometry of quartics X_4 with so-called lines of the second kind. Finally, I will explain what happens if we consider degree-4 surfaces in 3-dimensional affine space, allow the considered surface not to be a complex manifold in a finite set of points (resp. along a curve), replace the field of complex numbers with an algebraically closed field of positive characteristic. I will state some results for degree 5 (based on joint work with Prof. M. Schuett (LUH Hannover)).