

## INTRODUCTORY SPEECH

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The International Conference, “Satelite” to the Second European Congress of Mathematics in Budapest TOPOLOGICAL METHODS IN DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS is organized in the year of the 100th anniversary of the birth of Professor Tadeusz Ważewski born on September 24, 1896 a village in Włynianka.

After graduating from high school in Tarnów, Tadeusz Ważewski became in 1914 a student of the Jagiellonian University. He studied physics first, before changing his interest and switching to mathematics. He was a student of Stanisław Zaremba, an outstanding Polish analyst. Ważewski’s intuition gained during studying physics played an important role in his future activity as a teacher and researcher.

Ważewski spent the years 1921–1923 in Paris, where he got his Ph. D. (la grade de docteur és sciences mathématiques) in 1924; the thesis committee consisted of Émile Borel, Arnaud Denjoy and Paul Montel. He received his habilitation from the Jagiellonian University in 1927, becoming an extraordinary professor at this University in 1933 and being promoted to ordinary (full) professor in 1945. On November 6, 1939 he was, together with a large group of university professors, arrested by Nazi occupation forces in Poland and deported to a concentration camp. He was released before the end of the war and was acting at the underground University, teaching and continuing his research activity. He served as the President of the Polish Mathematical Society for the term 1957–1959, he was a member of the Polish Academy of Arts and Sciences and a member of the Polish Academy of Sciences. In 1967 he received the distinction of the honorary degree, that is the title of “doctor honoris causa” from the Jagiellonian University and the honorary membership of the Polish Mathematical Society.

Professor Tadeusz Ważewski died on September 5, 1972. The Polish Mathematical Society established a prize named after him.

First papers of Ważewski concerned some topological problems. For instance, his Ph. D. thesis [1] and habilitation dissertation [2] deal with connected continua not containing simple closed curves and with rectifiable continua, respectively.

Although Ważewski changed his major interest becoming an outstanding analyst, he never cut his ties with topology completely. Moreover, as it is well known, especially to all participants of this conference, some beautiful applications of topological methods permitted him to formulate and prove the famous Retract Theorem, which was the start point for future applications of advanced methods of algebraic topology in the theory of differential equations and general dynamical systems.

However, before the Retract Theorem was presented, Tadeusz Ważewski obtained – among others – important results concerning the existence and uniqueness of solutions of Cauchy problems for the first order partial differential equations, as well as theorems giving certain sharp estimations of domains of the existence of such solutions (see for instance [3]–[10]). Ważewski proved fundamental theorems on differential inequalities (see the paper [11] published after the war, but containing some results obtained during the war); these results inspired several other authors, among them Jacek Szarski, who summarized in some sense the theory in his book [21].

In the thirties S.K. Zaremba (the son of Stanisław Zaremba, who was the teacher of Ważewski) in Kraków and A. Marchaud in Paris considered (independently) generalizations of differential equations, called now differential inclusions (they used another terminology). Ważewski observed that the theory proposed by S.K. Zaremba and A. Marchaud can be applied – almost without any essential changes – to the optimal control theory, and developed this idea fruitfully. There are several papers written by Ważewski about that; we refer to one of them (as an example and a source for further references) [12].

Ordinary differential equations were subjects of several papers of Ważewski, mainly (but not only) in the context of the qualitative aspects of the theory. His observations on the method of successive approximations in several versions are very important.

Some important results concerning the “asymptotic coincidence” (the terminology of Ważewski) were proved in [13] and [14].

It is impossible to present here, in a brief introduction, all results of Ważewski, even all major results cannot be presented.

So I would like to close this collection of **selected** results of Ważewski by coming back to the Ważewski Retract Theorem and the Ważewski Topological Method. It would be superfluous to formulate them here and go into details.

So I will limit myself to cite only an opinion of Solomon Lefschetz, who said in 1961 that the retract method of Ważewski is one of the most important results in the theory of differential equations obtained after the World War II; (see [19]). Let us observe that Lefschetz did not know at that time (in 1961) how fruitful and stimulating this method would be for deep applications of really advanced methods of algebraic topology in differential equations and dynamical systems. There are several papers written by Ważewski on that subject; four of them are referred: [15]–[18].

Tadeusz Ważewski was a great scientist. He was also an outstanding teacher. He created around himself a group called the Kraków School of Differential Equations. There are 20 professors among his students. The late Jacek Szarski said in 1973 on the occasion of a special session devoted to the memory of his Master (and the Master of several of us being here in this hall): “Tadeusz Ważewski combined his own research with an intensive activity of teaching young mathematicians. He could discover mathematical talents even among first year students. Due to his very special didactic quality he was able to stimulate interest of his students... The question of didactics and methodology were always close to his heart and he devoted much attention and effort to these questions. The whole of his life is an example that research and teaching are strongly connected and should not be separated artificially” (see [19] and [20]).

References are added while adapting to this publications.

## References

A. Papers of Tadeusz Ważewski referred in the article.

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**B. Other references.**

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