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ON THE TORSION ON GAUGE-LIKE PROLONGATIONS OF PRINCIPAL BUNDLES

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Abstract. Every fiber product preserving bundle functor F on the category $\mathcal{FM}_m$ defines the gauge-like prolongation $W^F P$ of a principal bundle P, that coincides with the r-th principal prolongation $W^r P$ in the special case $F = J^r$. For a large class of such functors we introduce the torsion of connections on $W^F P$ and we deduce some of its properties analogous to the case of $W^r P$.

The r-th principal (or gauge-natural) prolongation $W^r P \to M$ of a principal bundle $P \to M$ is a fundamental structure for both the theory of geometric object fields [**2, 9**], and the gauge theories of mathematical physics [**3**]. In [**5**] we introduced the torsion of a connection Γ on $W^r P$ to be the covariant exterior differential of the canonical one-form θ_r of $W^r P$. On the other hand, the Lie algebroid $L(W^r P)$ coincides with the r-th jet prolongation $J^r(LP)$ of the Lie algebroid LP of P [**11**]. In [**8**], we considered the algebroid form $\gamma\colon TM \to J^r LP$ of Γ, we introduced the torsion of γ by using the truncated bracket of $J^r LP$ and we deduced that both approaches to the torsion are naturally equivalent.

In the present paper we study a more general setting of this problem. In [**1**] the authors constructed a principal bundle $W^F P \to M$ for every fiber product preserving bundle functor F on the category $\mathcal{FM}_m$ of fibered manifolds with m-dimensional bases and fibered morphisms with local diffeomorphisms as base maps. In the case of the functor J^r of r-th jet prolongation, we have $W^{J^r} P = W^r P$, so that $W^F P$ will be called a gauge-like prolongation of P. We are going

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to clarify that geometrically remarkable results concerning torsion appear in the case of a subfunctor $E \subset J^1 \circ F$. In this situation, we can use our results on the generalized G-structures on $W^F P$, i.e. the reductions of $W^1(W^F P)$ [**4**].

In Section 1 we first summarize the basic properties of $W^F P$. Then we construct a canonical map relating the gauge-like prolongation of the iteration of two functors with the iteration of the gauge-like prolongations. In Proposition 1 we determine the Lie algebroid version of this map. In Section 2 we study the reductions Q of $W^1 P$ (called generalized G-structures in [**4**]) from our point of view. In Proposition 2 we deduce that both approaches to the torsion on Q are naturally equivalent. Special attention is paid to the additional properties of semiprolongable generalized G-structures. Proposition 3 describes a relation between the prolongability of generalized G-structures and the existence of torsion-free connections analogous to the case of classical G-structures. In Section 3 we specify some fiber product preserving bundle functors on $\mathcal{FM}_m$, the gauge-like prolongations of which are of the form studied in Section 2. We also point out that some further functors can be reduced to this case by using a suitable natural equivalence.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notations from the book [**9**]. In particular, we write $P^r M$ for the r-th order frame bundle of a manifold M and G^r_m for the r-th jet group in dimension m. Under a connection we always mean a principal connection.

1. Gauge-like prolongations of principal bundles. We denote by T^A the Weil functor determined by a Weil algebra A [**9, 7**]. A fundamental result reads that the product preserving bundle functors on the category $\mathcal{M}f$ of all manifolds are in bijection with the Weil functors and the natural transformations $h_M \colon T^{A_1} M \to T^{A_2} M$ are in bijection with the algebra homomorphisms $h \colon A_1 \to A_2$, [**7**]. In the special case of $\mathbb{D}^r_k = J^r_0(\mathbb{R}^k, \mathbb{R})$, $T^{\mathbb{D}^r_k} = T^r_k$ is the classical functor of (k,r)-velocities.

In [**10**] the authors deduced that the fiber product preserving bundle functors on $\mathcal{FM}_m$ of base order r are in bijection with the triples (A, H, t) of a Weil algebra A, a group homomorphism $H \colon G^r_m \to \mathrm{Aut}\, A$ and an equivariant algebra homomorphism $t \colon \mathbb{D}^r_m \to A$, where $\mathrm{Aut}\, A$ is the group of all algebra automorphisms of A and we take into account $\mathrm{Aut}\,(\mathbb{D}^r_m) = G^r_m$. One constructs the functor $F = (A, H, t)$ as follows. For every manifold N, we have an induced action H_N of G^r_m on $T^A N$,

$$H_N(g, z) = H(g)_N(z)\,, \quad g \in G^r_m,\ z \in T^A N\,,$$

where $H(g)_N \colon T^A N \to T^A N$ is the map determined by $H(g) \colon A \to A$. The value of F on the product fibered manifold $M \times N \to M$, $\dim M = m$, is the associated bundle

$$F(M \times N) = P^r M[T^A N, H_N]\,. \tag{1}$$

Every local diffeomorphism $f\colon M \to \bar{M}$ and every map $\varphi\colon N \to \bar{N}$ determine the product $\mathcal{FM}_m$-morphism $f \times \varphi\colon M \times N \to \bar{M} \times \bar{N}$. By naturality, the map $T^A\varphi\colon T^A N \to T^A \bar{N}$ is G^r_m-equivariant. Further, $P^r f\colon P^r M \to P^r \bar{M}$ is a principal bundle morphism. Then we define $F(f\times\varphi)\colon F(M\times N) \to F(\bar{M}\times\bar{N})$ to be the morphism of associated bundles

$$F(f \times \varphi) = (P^r f, T^A\varphi)\colon P^r M[T^A N, H_N] \to P^r \bar{M}[T^A \bar{N}, H_{\bar{N}}]\,. \tag{2}$$

For an arbitrary fibered manifold $p\colon Y \to M$, FY is the subbundle of $P^r M[T^A Y, H_Y]$ of all elements $\{u, Z\}$, $u \in P^r M \subset T^r_m M$, $Z \in T^A Y$ satisfying

$$t_M(u) = T^A p(Z)\,, \tag{3}$$

where $t_M\colon T^r_m M \to T^A M$ is the map induced by $t\colon \mathbb{D}^r_m \to A$. Since t is equivariant, (3) is independent of the choice of the representatives u and Z of the equivalence class $\{u, Z\} \in FY$. For another fibered manifold $\bar{p}\colon \bar{Y} \to \bar{M}$ and an $\mathcal{FM}_m$-morphism $f\colon Y \to \bar{Y}$ over $\underline{f}\colon M \to \bar{M}$, $(P^r \underline{f}, T^A f)$ maps FY into $F\bar{Y}$. Then one defines Ff to be its restriction and corestriction. In the case of J^r, we have $A = \mathbb{D}^r_m$, $H = \mathrm{id}_{G^r_m}$, $t = \mathrm{id}_{\mathbb{D}^r_m}$ and (3) expresses, in fact, the classical relation

$$J^r Y = \big\{X \in J^r(M, Y);\ p_*(X) = j^r_x \,\mathrm{id}_M\big\}\,, \tag{4}$$

where x is the source of $X = j^r_x f$ and $p_*(X) = j^r_x(p \circ f) \in J^r(M, M)$.

Consider a principal bundle $P(M, G)$. First we recall $W^r P = P^r M \times_M J^r P$, [**9**]. This is a principal bundle over M, whose structure group is the group semidirect product $W^r_m G = G^r_m \rtimes T^r_m G$ with the composition

$$(g_1, C_1)(g_2, C_2) = \big(g_1 \circ g_2, (C_1 \circ g_2) \bullet C_2\big)\,, \tag{5}$$

where $\bullet$ denotes the induced group composition in $T^r_m G$. In [**1**] the authors defined

$$W^F P = P^r M \times_M FP\,. \tag{6}$$

Analogously to (5), one constructs the group semidirect product

$$W^A_H G = G^r_m \rtimes_H T^A G$$

with the composition

$$(g_1, C_1)(g_2, C_2) = \big(g_1 \circ g_2, H(g_2^{-1})_G(C_1) \bullet C_2\big)\,, \tag{7}$$

where $\bullet$ denotes the induced group composition in $T^A G$. In the case of $W^r_m G$, $H(g_2^{-1})_G(C_1) = C_1 \circ g_2$, so that (7) generalizes (5). Then $W^F P$ is a principal bundle over M with structure group $W^A_H G$ with respect to the following action [**1**]. The right action of G on P can be interpreted as an $\mathcal{FM}_m$-morphism

$$\varrho\colon P \times_M (M \times G) \to P\,.$$

Applying F, we obtain

$$F\varrho\colon FP \times_M P^r M[T^A G, H_G] \to FP\,.$$

For $(g, X) \in G^r_m \times T^A G$ and $(u, Y) \in P^r M \times_M FP$, one defines

$$(u, Y)(g, X) = \bigl(u \circ g, F\varrho(Y, \{u \circ g, X\})\bigr)\,. \tag{8}$$

In the case of $W^r P$, (8) coincides with the right action of $W^r_m G$ on $W^r P$ described in [**9**].

We remark that a basic geometric property of $W^F P$ is that for every bundle $D \to M$ associated to P, $FD \to M$ is a bundle associated to $W^F P$ [**1**].

Further, F determines a natural transformation $\tilde{t}_Y\colon J^r Y \to FY$. Every element $X \in J^r Y$ is of the form $j^r_x s$. We interpret the local section s of Y as a local $\mathcal{FM}_m$-morphism $\tilde{s}$ of the trivial fibered manifold $\mathrm{id}_M\colon M \to M$ into Y and we set

$$\tilde{t}_Y(X) = (F\tilde{s})(x) \in FY\,. \tag{9}$$

In the product case, $J^r(M \times N) = P^r M[T^r_m N]$, $F(M \times N) = P^r M[T^A N]$ and $\tilde{t}_{M\times N}$ is of the form $\tilde{t}_{M\times N} = (\mathrm{id}_{P^r M}, t_N)$ with $t_N\colon T^r_m N \to T^A N$.

In particular, we have $\tilde{t}_{TM}\colon J^r TM \to FTM$. Write $q\colon LP \to TM$ for the anchor map, so that $Fq\colon FLP \to FTM$. In [**6**] we deduced that the Lie algebroid of $W^F P$ is

$$L(W^F P) = J^r TM \times_{FTM} FLP\,. \tag{10}$$

In the special case $F = J^r$, we reobtain

$$L(W^r P) = J^r TM \times_{J^r TM} J^r LP = J^r LP\,.$$

Consider two such functors F_1 and F_2 of base orders r and s. Then the base order of $F_2 \circ F_1$ is $r + s$. Using $(\tilde{t}_2)_{P^r M}\colon J^s P^r M \to F_2 P^r M$, we construct a map

$$W^{F_2 \circ F_1} P \to W^{F_2}(W^{F_1} P) \tag{11}$$

as follows. The classical inclusion $P^{r+s} M \hookrightarrow W^s(P^r M) = P^s M \times_M J^s P^r M$ is described in [**9**]. So we have

$$\begin{aligned} W^{F_2\circ F_1} P = P^{r+s} M \times_M F_1 F_1 P &\hookrightarrow P^s M \times_M J^s P^r M \times_M F_2 F_1 P \\ &\to P^s M \times_M F_2 P^r M \times_M F_2 F_1 P \\ &\to P^s M \times_M F_2(W^{F_1} P) = W^{F_2}(W^{F_1} P)\,. \end{aligned} \tag{12}$$

According to [**1**], this is a principal bundle morphism. If t_2 is injective, (11) is an injection.

To construct the corresponding Lie algebroid homomorphism, we start with a general formula. Consider two principal bundles $P_1 \to M_1$ and $P_2 \to M_2$ over m-manifolds and a $\mathcal{PB}$-morphism $f\colon P_1 \to P_2$ over a local diffeomorphism $\underline{f}\colon M_1 \to M_2$. Write $Lf\colon LP_1 \to LP_2$ for the induced algebroid morphism. Using trivializations, one finds easily that the algebroid morphism $LW^F f\colon LW^F P_1 \to LW^F P_2$ is

$$J^rT\underline{f} \times_{FT\underline{f}} FLf\colon J^rTM_1 \times_{FTM_1} FLP_1 \to J^rTM_2 \times_{FTM_2} FLP_2\,. \tag{13}$$

Further, the algebroid form of the injection $P^{r+s}M \hookrightarrow W^sP^rM$ is

$$J^{r+s}TM \hookrightarrow J^sTM \times_{J^sTM} J^sJ^rTM\,,$$

where we consider both the jet projection $\pi^{r+s}_s\colon J^{r+s}TM \to J^sTM$ and the canonical injection $J^{r+s}TM \hookrightarrow J^sJ^rTM$. According to [**1**], $(\tilde{t}_2)_{P^rM}\colon J^sP^rM \to F_2P^rM$ induces a principal bundle morphism

$$W^sP^rM = P^sM \times_M J^sP^rM \to P^sM \times_M F_2P^rM = W^{F_2}P^rM\,.$$

One verifies easily that its algebroid form

$$J^sTM \times_{J^sTM} J^sJ^rTM \to J^sTM \times_{F_2TM} F_2J^rTM \tag{14}$$

is determined by $(\tilde{t}_2)_{J^rTM}\colon J^sJ^rTM \to F_2J^rTM$. So we have

$$\begin{aligned} L(W^{F_2\circ F_1}P) &= J^{r+s}TM \times_{F_2F_1TM} F_2F_1LP \\ &\hookrightarrow J^sJ^rTM \times_{F_2F_1TM} F_2F_1LP \\ &\to (J^sTM \times_{F_2TM} F_2J^rTM) \times_{F_2F_1TM} F_2F_1LP \\ &= J^sTM \times_{F_2TM} F_2(LW^{F_1}P) = L\big(W^{F_2}(W^{F_1}P)\big)\,. \end{aligned} \tag{15}$$

Using (12)–(15), we deduce

Proposition 1. *The algebroid homomorphism*

$$L(W^{F_2\circ F_1}P) \to L\big(W^{F_2}(W^{F_1}P)\big) \tag{16}$$

corresponding to (11) *is the composition of all arrows in* (15).

2. Reductions of W^1P. First we summarize our results from [**8**] on the torsion of connections on W^rP in the case $r = 1$. We consider the principal bundle $W^1P = P^1M \times_M J^1P$ with structure group $W^1_mG = G^1_m \rtimes T^1_mG$ and the canonical one-form $\theta_1\colon TW^1P \to \mathbb{R}^m \times \mathfrak{g}$. For a connection Γ on W^1P, the torsion is defined to be the covariant exterior differential $D_\Gamma\theta_1$. This can be interpreted as a map

$$\{D_\Gamma\theta_1\}\colon W^1P \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}\,. \tag{17}$$

On the other hand, we have $L(W^1P) = J^1(LP)$. Since the bracket $[\![\,,\,]\!]$ of LP is a first order differential operator, it induces the so-called truncated bracket

$$[\![\,,\,]\!]_1\colon J^1LP \times_M J^1LP \to LP$$

[**8**]. If we pass to the algebroid form $\gamma\colon TM \to J^1LP$ of Γ, we introduce $\tau\gamma\colon TM \times_M TM \to LP$ by

$$\tau\gamma(Z_1, Z_2) = [\![\gamma Z_1, \gamma Z_2]\!]_1\,, \qquad (Z_1, Z_2) \in TM \times_M TM\,. \tag{18}$$

This is a section of $LP \otimes \Lambda^2 T^*M$, what is a fiber bundle associated to W^1P with standard fiber $(\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}$. So the frame form of $\tau\gamma$ is a map

$$\{\tau\gamma\}\colon W^1P \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}\,. \tag{19}$$

In [**8**] we deduced

$$\{D_\Gamma\theta_1\} = \frac{1}{2}\{\tau\gamma\}\,. \tag{20}$$

Remark. The coordinate formula for $\tau\gamma$ can be found in [**8**]. We find remarkable that this formula implies directly the following assertion. If Γ_1 and Γ_2 are two torsion-free connections on W^1P over the same connection on P, then every connection of the pencil $t\Gamma_1 + (1-t)\Gamma_2$, $t \in \mathbb{R}$, is also torsion-free.

Consider a reduction $Q \subset W^1P$ to a subgroup $H \subset W^1_mG$. In [**4**] Q is said to be a generalized G-structure. Write $\theta_Q\colon TQ \to \mathbb{R}^m \times \mathfrak{g}$ or $[\![\,,\,]\!]_Q\colon LQ \times_M LQ \to LP$ for the restriction of θ_1 or $[\![\,,\,]\!]_1$, respectively, and $i_Q\colon Q \to W^1P$ for the injection. A connection Γ on Q is canonically extended into a connection $\bar\Gamma$ on W^1P. Clearly, we have

$$D_\Gamma\theta_Q = i_Q^*(D_{\bar\Gamma}\theta_1)\,. \tag{21}$$

Further, for the algebroid form $\gamma\colon TM \to LQ$ of Γ, we define

$$\tau\gamma(Z_1, Z_2) = [\![\gamma Z_1, \gamma Z_2]\!]_Q\,, \qquad (Z_1, Z_2) \in TM \times_M TM\,.$$

Analogously to (19), we have its frame form

$$\{\tau\gamma\}\colon Q \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}$$

satisfying

$$\{\tau\gamma\} = \{\tau\bar\gamma\} \circ i_Q\,. \tag{22}$$

Then (20)–(22) imply

Proposition 2. *Both approaches to the torsion of connections on Q are related by*

$$\{D_\Gamma\theta_Q\} = \frac{1}{2}\{\tau\gamma\}\colon Q \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}\,. \tag{23}$$

Further we need the basic facts concerning the prolongation of generalized G-structures. We define the second nonholonomic prolongation $\tilde{W}^2P = W^1(W^1P)$. By [**4**], $\tilde{W}^2P = \tilde{P}^2M \times_M \tilde{J}^2P$, where $\tilde{P}^2M$ is the second nonholonomic frame bundle of M and $\tilde{J}^2P = J^1(J^1P)$. The second semiholonomic prolongation $\bar{W}^2P \subset \tilde{W}^2P$ can be defined as $\bar{P}^2M \times_M \bar{J}^2P$. We have $W^1Q \subset \tilde{W}^2P$ and $\beta\colon W^1Q \to Q$ is always surjective. According to [**4**], Q is called semiprolongable or prolongable, if the restriction of β to $W^1Q \cap \bar{W}^2P$ or $W^1Q \cap W^2P$ is also surjective, respectively. Write $Q_0 = \beta Q$ and $H_0 = \beta H$. In [**4**] we deduced

Lemma. *Q is semiprolongable, if and only if $Q \subset W^1(Q_0)$ or, equivalently, the values of θ_Q lie in $\mathbb{R}^m \times \mathfrak{h}_0$.*

Thus, if Q is semiprolongable, then (23) holds with the additional property that the values lie in $(\mathbb{R}^m \times \mathfrak{h}_0) \otimes \Lambda^2\mathbb{R}^{m*}$.

For every $X \in W^1P$, $X = (u, U)$, we have $U \circ u \in T^1_m P$. Hence X is identified with an m-dimensional subspace $\lambda(X) \subset TP$. So every $Z \in \tilde{W}^2P$ is identified with $\lambda(Z) \subset TW^1P$. According to Proposition 5 of [**4**], $Z \in \bar{W}^2P$ satisfies

$$(24) \qquad Z \in W^2P \quad \text{if and only if} \quad d\theta_1 \mid \lambda(Z) = 0\,.$$

This implies an assertion analogous to the classical theory of G-structures. Write $\pi\colon Q \to M$, $\pi_1\colon Q \to P^1M$ and $\pi_2\colon W^1Q \to J^1Q$ for the canonical projections.

Proposition 3. *Let Q be semiprolongable. If Q admits a torsion-free connection, then Q is prolongable. Conversely, if Q is prolongable, then for every $x \in M$ there exists a neighbourhood U and a torsion-free connection on $\pi^{-1}(U)$.*

Proof. Let $\Gamma\colon Q \to J^1Q$ be a torsion-free connection. By (24), the rule

$$X \mapsto \big(\pi_1(X), \Gamma(X)\big)\,, \qquad X \in Q$$

is a section $Q \to W^1Q \cap W^2P$, so that Q is prolongable. Conversely, let $\Sigma\colon Q \to W^1Q \cap W^2P$ be a section. For every section $\varrho\colon U \to Q$, the map $\pi_2 \circ \Sigma \circ \varrho\colon U \to J^1Q$ is canonically extended into a connection on $\pi^{-1}(U)$. This connection is torsion-free due to the fact that θ_1 is a pseudo-tensorial form [**9**, p. 155]. $\square$

3. Torsions on certain gauge-like prolongations. If E is a fiber product preserving bundle functor on $\mathcal{FM}_m$ satisfying $E \subset J^1 \circ F$, then W^EP is a reduction of $W^1(W^FP)$ to a subgroup $K \subset W^1_m(W^A_HG)$. Write $E_0 = \beta E \subset F$ and $K_0 = \beta K \subset W^A_HG$. In general, we have $\theta_{EP}\colon T(EP) \to \mathbb{R}^m \times \mathfrak{w}^A_HG$. If

EP is semiprolongable, then the values of θ_{EP} lie in $\mathbb{R}^m \times \mathfrak{k}_0$. The simpliest case of such situation is $\beta E = F$. We are going to present some examples.

Example 1. We start with the functor $\tilde{J}^r$ of r-th nonholonomic jet prolongation of fibered manifolds, $\tilde{J}^r Y = J^1(\tilde{J}^{r-1} Y)$. More generally, our approach can be applied to an arbitrary functor S of r-th jet prolongation of fibered manifolds. In accordance with [**7**], this means a fiber product preserving bundle functor on $\mathcal{FM}_m$ satisfying $J^r \subset S \subset \tilde{J}^r$. In particular, J^r can be reduced to this situation by means of the canonical inclusion $J^r \subset J^1 \circ J^{r-1}$. A further well known example is the r-th semiholonomic prolongation $\bar{J}^r \subset J^1 \circ \bar{J}^{r-1}$. Clearly, the semiprolongability condition is satisfied in the last two cases.

Example 2. Another example of our type is the composition $J^r \circ F$ for arbitrary F. More generally, we can consider every composition $S \circ F$ with S from Example 1.

Some further functors can be studies in this way by using a suitable natural equivalence.

Example 3. In [**1**] it is deduced that for every fiber product preserving bundle functor E on $\mathcal{FM}_m$ and every vertical Weil functor V^B there exists a canonical natural equivalence

$$\varkappa \colon V^B \circ E \approx E \circ V^B \,. \tag{25}$$

If $E \subset J^1 \circ F$, then $\varkappa(V^B \circ E) \subset J^1 \circ (F \circ V^B)$. Hence we have the situation of Section 2. In addition, one deduces that $\beta E = F$ implies $\beta\big(\varkappa(V^B \circ E)\big) = F \circ V^B$ by using the standard Weil algebra manipulations from [**1**].

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