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# WEAK* CONVERGENCE OF THE COMPLEX MONGE–AMPÈRE MEASURES ASSOCIATED WITH CERTAIN CLASSES OF DELTA-PLURISUBHARMONIC FUNCTIONS

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**Abstract.** The aim of this paper is to investigate the weak* convergence of the complex Monge–Ampère measures of delta-plurisubharmonic functions convergent in $\mathrm{C}_n$-capacity and in $\mathrm{C}_T$-capacity for a closed positive current $T$.

**1. Introduction.** The weak* convergence of sequences of the complex Monge–Ampère measures of plurisubharmonic functions convergent in capacity has been studied by many authors. In 1996 Y. Xing established the weak* convergence of the complex Monge–Ampère measures of locally bounded plurisubharmonic functions convergent in $\mathrm{C}_{n-1}$-capacity or $\mathrm{C}_n$-capacity (see [**14**]). After that in [**15**] he extended the above results for plurisubharmonic functions with bounded values near the boundary. The generalization of the above results onto some classes of plurisubharmonic functions on which the complex Monge–Ampère measures are well defined has recently been completed by Cegrell. In [**5**] Cegrell proved the following result. If $u_j, u \in \mathcal{E}(\Omega)$, $u_j, u$ are uniformly bounded from below by a function from $\mathcal{F}(\Omega)$ and if $u_j \longrightarrow u$ in capacity, then $(dd^c u_j)^n$ is weak* convergent to $(dd^c u)^n$. In Theorem 3.2 below, we extend Cegrell's above result by replacing the class $\mathcal{E}$ by the class $\delta\mathcal{E}_{loc}(\Omega)$. At the same time, we also study the weak* convergence of currents of the form $(dd^c u_j)^p \wedge T$ , where $T$ is a closed positive current of bidimension $(p,p)$ and $u_j$ are delta-plurisubharmonic functions convergent in $\mathrm{C}_T$-capacity.

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The paper is organized as follows. In Section 2 we recall some background of pluripotential theory presented in the paper by E. Bedford and B. A. Taylor, as well as in the book by M. Klimek (see [**1**,**10**]). We also deal with some classes of plurisubharmonic functions introduced and investigated by U. Cegrell in [**4**]. At the same time, we recall the two classes of delta-plurisubharmonic functions, the class of $\delta^*\mathrm{PSH}_{loc}$ in [**9**] and the class of $\delta\mathcal{E}_{loc}(\Omega)$ introduced in [**11**]. The main results of the paper are stated and proved in Section 3.

**2. Background.** In this section we recall some results about delta-plurisubharmonic functions and their Monge–Ampère measures, the capacity of a Borel set in the sense of Bedford and Taylor, as well as the capacity associated to a closed positive current $T$.

**2.1** Let $\Omega$ be an open set in $\mathbb{C}^n$. We say that $u \in \delta^*\mathrm{PSH}_{loc}(\Omega)$ if for each z in $\Omega$ there exist a neighbourhood $U$ of z in $\Omega$ and two plurisubharmonic functions $v_1, v_2 \in \mathrm{PSH}(U) \cap \mathrm{L}^\infty(U)$ such that

$$u = v_1 - v_2$$

on $U$.

In [**9**], the authors have proved that if $u \in \delta^*\mathrm{PSH}_{loc}(\Omega)$ and $\{U_i\}_{i\geq 1}$ is an open covering of $\Omega$ such that $u = v_{i,1} - v_{i,2}$ on $U_i$ for $i \geq 1$, where $v_{i,1}, v_{i,2} \in \mathrm{PSH}(U_i) \cap \mathrm{L}^\infty(U_i)$ and $0 \leq m \leq n$, then on $U_i$ we have

$$(dd^c u)^m = \sum_{k=0}^{m} (-1)^k \binom{m}{k} (dd^c v_{i,1})^k \wedge (dd^c v_{i,2})^{m-k}$$

and, hence, $(dd^c u)^m$ is a closed current of bidegree $(m, m)$ on $\Omega$ (see Proposition–Definition 2.2 in [**9**]). Moreover, they have shown that the above definition does not depend on the choice of the open covering $\{U_i\}$.

Based on the above definition we give the following. Let $u \in \delta^*\mathrm{PSH}_{loc}(\Omega)$ and $\{U_i\}_{i\geq 1}$ be an open covering of $\Omega$ such that $u = v_{i,1} - v_{i,2}$ on $U_i$ for $i \geq 1$, where $v_{i,1}, v_{i,2} \in \mathrm{PSH}(U_i) \cap \mathrm{L}^\infty(U_i)$. Let $T$ be a closed positive current of bidimension $(p, p)$ on $\Omega$, $0 \leq p \leq n$. Then on each $U_i$ we can define a signed regular Borel measure

$$(dd^c u)^p \wedge T = \sum_{k=0}^{p} (-1)^k \binom{p}{k} (dd^c v_{i,1})^k \wedge (dd^c v_{i,2})^{p-k} \wedge T.$$

By the same arguments as in [**9**], we note that $(dd^c u)^p \wedge T$ is a signed regular Borel measure on $\Omega$.

**2.2.** Let $\Omega \subset \mathbb{C}^n$ be an open set and $E \subset \Omega$ a Borel subset. The $\mathrm{C}_n$-capacity of $E$, introduced by Bedford and Taylor in [**1**], is defined by

$$\mathrm{C}_n(E) = \mathrm{C}_n(E,\Omega) = \sup\Big\{ \int\limits_E (dd^c u)^n : u \in \mathrm{PSH}(\Omega), \quad 0 \le u \le 1 \Big\}.$$

We state the following results on $\mathrm{C}_n$-capacity (see [**2**], [**13**]).
**2.2.1.** If $E_1 \subset E_2 \subset \Omega_1 \subset \Omega_2$ then $\mathrm{C}_n(E_1,\Omega_2) \le \mathrm{C}_n(E_2,\Omega_1)$.
**2.2.2.** $\mathrm{C}_n(\bigcup\limits_{j=1}^{\infty} E_j) \le \sum\limits_{j=1}^{\infty} \mathrm{C}_n(E_j)$.
**2.2.3.** If $E_j \uparrow E$, then $\lim\limits_{j\to\infty} \mathrm{C}_n(E_j) = \mathrm{C}_n(E)$.

**2.3.** Recently Dabekk and Elkhadhra introduced the notion of a capacity associated with a closed positive current $T$ of bidimension (p,p) on an open set $\Omega$ of $\mathbb{C}^n$. Let $\Omega$ be an open set in $\mathbb{C}^n$ and $E \subset \Omega$ be a Borel set. Let $T$ be a closed positive current of bidimension (p,p), $p \ge 1$ on $\Omega$. The capacity of $E$ with respect to $\Omega$, denoted by $\mathrm{C}_T(E,\Omega) = \mathrm{C}_T(E)$, is defined by

$$\mathrm{C}_T(E,\Omega) = \mathrm{C}_T(E) = \sup\Big\{ \int\limits_E T \wedge (dd^c v)^p : v \in \mathrm{PSH}(\Omega), \quad 0 < v < 1 \Big\}.$$

Similarly to $\mathrm{C}_n$-capacity, $\mathrm{C}_T$-capacity has the following properties.
**2.3.1.** If $E_1 \subset E_2 \subset \Omega_1 \subset \Omega_2$ then $\mathrm{C}_T(E_1,\Omega_2) \le \mathrm{C}_T(E_2,\Omega_1)$.
**2.3.2.** If $E_1, E_2, \cdots$ are Borel subsets of $\Omega$, then

$$\mathrm{C}_T(\bigcup_{j=1}^{\infty} E_j) \le \sum_{j=1}^{\infty} \mathrm{C}_T(E_j).$$

**2.3.3.** If $E_1 \subset E_2 \subset \cdots$ are Borel subsets of $\Omega$, then

$$\mathrm{C}_T(\bigcup_{j=1}^{\infty} E_j) = \lim_{j\to\infty} \mathrm{C}_T(E_j).$$

(See Definition 2.1 and Proposition 2.2 in [**8**] for details).

**2.4.** Now we recall definitions of convergence in $\mathrm{C}_n$-capacity and $\mathrm{C}_T$-capacity (see [**8**], [**14**]).

Let $\{u_j\}_{j\ge1}$ and $u$ be functions on an open set $\Omega \subset \mathbb{C}^n$ and $E \subset \Omega$. We say that the sequence $\{u_j\}$ is convergent to $u$ in $\mathrm{C}_n$-capacity ( resp., in $\mathrm{C}_T$-capacity) on $E$ if for all $\delta > 0$, there is

$$\lim_{j\to\infty} \mathrm{C}_n\Big(\Big\{ z \in E : |u_j(z) - u(z)| > \delta \Big\}\Big) = 0.$$

(Resp., $\lim\limits_{j\to\infty} \mathrm{C}_T\Big(\Big\{ z \in E : |u_j(z) - u(z)| > \delta \Big\}\Big) = 0.$

**2.5.** Let $\mu_n, \mu$ be Borel measures on an open set $\Omega \subset \mathbb{C}^n$. We say that the sequence $\{\mu_n\}_{n\geq 1}$ is weak* convergent to $\mu$ if

$$\int\limits_{\Omega} \phi d\mu_n \longrightarrow \int\limits_{\Omega} \phi d\mu \quad \text{for all } \phi \in C_0^\infty(\Omega),$$

where $C_0^\infty(\Omega)$ denotes the set of smooth functions with compact support on $\Omega$.

**2.6** Let $\mu$ be a Borel measure on an open set $\Omega \subset \mathbb{C}^n$. We say that $\mu$ is absolutely continuous with respect to $C_n$-capacity if for each $\varepsilon > 0$ there exists $\delta > 0$ such that for every Borel set $E \subset \Omega$ with $C_n(E) < \delta$ there follows that $\mu(E) < \varepsilon$. It is easy to see that $\mu$ is absolutely continuous with respect to $C_n$-capacity if and only if it vanishes on all pluripolar sets $F \subset \Omega$. Here a subset $F \subset \Omega$ is pluripolar if there exists a plurisubharmonic function $\varphi$ on $\Omega$, $\varphi \not\equiv -\infty$, such that $F \subset \{z \in \Omega : \varphi(z) = -\infty\}$. Indeed, the necessity is obvious. To prove the sufficiency, we assume that $\mu$ vanishes on all pluripolar sets $E \subset \Omega$ but $\mu$ is not absolutely continuous with respect to $C_n$-capacity. Then there exist $\varepsilon_0 > 0$ and a decreasing sequence of Borel sets $\{E_k\}_{k\geq 1} \subset \Omega$ such that

$$C_n(E_k) < \frac{1}{k}, \quad k = 1, 2, \dots,$$

and

$$\mu(E_k) \geq \varepsilon_0 \quad \forall \quad k \geq 1.$$

Put $E = \bigcap\limits_{k=1}^{\infty} E_k$. Then $C_n(E) = 0$ and, by Theorem 6.9 in [**1**], $E$ is pluripolar. But $\mu(E) = \lim\limits_k \mu(E_k) \geq \varepsilon_0$ and we get a contradiction.

**2.7.** Now we deal with the following classes $\mathcal{E}_0, \mathcal{F}$ and $\mathcal{E}$ of plurisubharmonic functions introduced and investigated by Cegrell in [**4**]. We introduce the class $\delta\mathcal{E}_{loc}(\Omega)$.

Let $\Omega$ be a bounded hyperconvex domain in $\mathbb{C}^n$. Then

$$\mathcal{E}_0 = \mathcal{E}_0(\Omega) = \{\varphi \in \mathrm{PSH}(\Omega) \cap \mathrm{L}^\infty(\Omega) : \lim_{z\to\partial\Omega} \varphi(z) = 0, \ \int\limits_{\Omega} (dd^c\varphi)^n < \infty\},$$

$$\mathcal{F} = \mathcal{F}(\Omega) = \big\{\varphi \in \mathrm{PSH}(\Omega) : \exists \ \mathcal{E}_0 \ni \varphi_j \searrow \varphi, \ \sup_j \int\limits_{\Omega} (dd^c\varphi_j)^n < \infty\big\},$$

$$\mathcal{E} = \mathcal{E}(\Omega) = \big\{\varphi \in \mathrm{PSH}(\Omega) : \forall z_0 \in \Omega, \text{there is a neighbourhood } \ \omega \ni z_0,$$
$$\mathcal{E}_0 \ni \varphi_j \searrow \varphi \ \text{ on } \ \omega, \ \sup_j \int\limits_{\Omega} (dd^c\varphi_j)^n < \infty\big\}.$$

The following inclusions are clear: $\mathcal{E}_0 \subset \mathcal{F} \subset \mathcal{E}$.

It follows from [**4**] that if $u \in \mathcal{E}$ then $(dd^c u)^n$ is well defined and it is a positive Radon measure on $\Omega$. Moreover, Błocki in [**3**] proved that $\mathcal{E}$ has the local property, i.e., $u \in \mathcal{E}(\Omega)$ if and only if for each $z \in \Omega$ there is a neighbourhood $U_z$ of $z$ in $\Omega$ such that $u \in \mathcal{E}(U_z)$. Notice that Theorem 4.5 in [**4**] implies that every $u \in \mathcal{E}(\Omega)$ is locally in $\mathcal{F}(\Omega)$. Now we introduce the class $\delta\mathcal{E}_{loc}(\Omega)$. We say that $u \in \delta\mathcal{E}_{loc}(\Omega)$ if for each $z \in \Omega$ there is a neighbourhood $U_z$ of $z$ in $\Omega$ such that $u = v - w$ on $U_z$, where $v, w \in \mathcal{E}(U_z)$. As in [**11**] if $u \in \delta\mathcal{E}_{loc}(\Omega)$ then $(dd^c u)^n$ is well defined and it is a signed Borel measure on $\Omega$.

**3. Results.** The first result of this section is the following.

THEOREM 3.1. *Let $\Omega$ be an open set in $\mathbb{C}^n$ and $T$ be a closed positive current of bidimension $(p,p)$ on $\Omega$ and $\{u_j\} \subset \delta^* PSH_{loc}(\Omega)$ and $u$ a $\delta$-plurisubharmonic function on $\Omega$ in the sense $u = v - w$, where $v, w$ are locally bounded plurisubharmonic functions on $\Omega$. Assume that*

*i) $u_j \longrightarrow u$ in $C_T$-capacity on every $E \Subset \Omega$.*

*ii) For all $z \in \Omega$, there is a neighbourhood $U_z$ of $z$ in $\Omega$ such that for all $j \geq 1$, $u_j = v_j^1 - v_j^2$, where $v_j^1, v_j^2$ are uniformly bounded plurisubharmonic functions on $U_z$ for all $j \geq 1$.*

*Then $(dd^c u_j)^p \wedge T$ is weak* convergent to $(dd^c u)^p \wedge T$.*

PROOF. First we prove the theorem for $p = 1$. Namely we prove that

$$\lim_{j\to\infty} \int_\Omega \psi dd^c u_j \wedge T = \int_\Omega \psi dd^c u \wedge T \tag{3.1}$$

for all $\psi \in C_0^\infty(\Omega)$. Let $\psi \in C_0^\infty(\Omega)$ be given. There holds

$$\begin{aligned}
&\Big|\int_\Omega \psi dd^c u_j \wedge T - \int_\Omega \psi dd^c u \wedge T\Big| = \Big|\int_\Omega \psi dd^c (u_j - u) \wedge T\Big| \\
&= \Big|\int_\Omega (u_j - u) dd^c\psi \wedge T\Big| = \Big|\int_K (u_j - u) dd^c\psi \wedge T\Big| \\
&= \Big|\int_{K\cap\{|u_j-u|>\varepsilon\}} (u_j - u) dd^c\psi \wedge T + \int_{K\cap\{|u_j-u|\leq\varepsilon\}} (u_j - u) dd^c\psi \wedge T\Big| \\
&\leq \int_{K\cap\{|u_j-u|>\varepsilon\}} |u_j - u| \|dd^c\psi \wedge T\| + \int_{K\cap\{|u_j-u|\leq\varepsilon\}} |u_j - u| \|dd^c\psi \wedge T\|,
\end{aligned} \tag{3.2}$$

where $\varepsilon > 0$ is given, $K = \operatorname{supp}\psi$ and $\|dd^c\psi \wedge T\|$ denotes the total variation of the regular Borel signed measure $dd^c\psi \wedge T$.

Proposition 3.2.7 in [**10**] implies that there exists a positive constant $\mathrm{C} = \mathrm{C}(n,1)$ such that

$$\omega = \mathrm{C}\|dd^c\psi\|\beta + dd^c\psi$$

is $(1,1)$ nonnegative real form, where $\beta = \frac{i}{2}\sum\limits_{j=1}^{n} dz_j \wedge d\bar{z}_j$ is the canonical Kähler form of $\mathbb{C}^n$. Hence,

$$dd^c\psi \wedge T = T \wedge \omega - \mathrm{C}\|dd^c\psi\| T \wedge \beta.$$

Then

$$\|dd^c\psi \wedge T\| \leq T \wedge \omega + \mathrm{C}\|dd^c\psi\| T \wedge \beta. \tag{3.3}$$

Lemma 1.3.8 in [**2**] implies that

$$T \wedge \omega + \mathrm{C}\|dd^c\psi\| T \wedge \beta \leq \mathrm{D} \quad T \wedge \beta,$$

where D is a constant depending only on $\psi$. Hence, the right-hand side of (3.2) does not exceed

$$(3.2) \leq \mathrm{D}\Big[\int\limits_{K\cap\{|u_j-u|>\varepsilon\}} |u_j - u| T \wedge \beta + \int\limits_{K\cap\{|u_j-u|\leq\varepsilon\}} |u_j - u| T \wedge \beta\Big].$$

First, we notice that

$$\int\limits_{K\cap\{|u_j-u|\leq\varepsilon\}} |u_j - u|\, T \wedge \beta \leq \varepsilon \int\limits_{K\cap\{|u_j-u|\leq\varepsilon\}} T \wedge \beta \leq \varepsilon \int\limits_{K} T \wedge \beta.$$

Secondary,

$$\begin{aligned}\int\limits_{K\cap\{|u_j-u|>\varepsilon\}} |u_j - u|\, T \wedge \beta &\leq \int\limits_{K\cap\{|u_j-u|>\varepsilon\}} (|u_j| + |u|) \quad T \wedge \beta \\ &\leq \mathrm{M}(K,u) \int\limits_{K\cap\{|u_j-u|>\varepsilon\}} T \wedge \beta \\ &\leq \mathrm{M}_1(K,u)\mathrm{C}_T\Big(K \cap \{|u_j - u| > \varepsilon\}, \Omega\Big) \longrightarrow 0\end{aligned}$$

as $j \longrightarrow \infty$, where $\mathrm{M}(K,u)$ and $\mathrm{M}_1(K,u)$ are constants depending on $K$ and u only.

Hence, we have proved (3.1).

Now assume that the theorem holds for $p = s, 1 \leq s \leq n-1$. We show that it is true for $s+1$. It suffices to show that

$$u_j T \wedge (dd^c u_j)^s \longrightarrow uT \wedge (dd^c u)^s$$

in weak* topology. Indeed, by the hypothesis we can write $u = v - w$, where $v, w \in \mathrm{PSH}(\Omega) \cap \mathrm{L}^\infty(\Omega)$. Theorem 2.5 in [**8**] implies that for $\varepsilon > 0$ there exists an open subset $G \subset \Omega$, $\mathrm{C}_T(G,\Omega) < \varepsilon$ such that $v = v_1 + \psi_1, w = w_1 + \psi_2$, where $v_1, w_1$ are continuous on $\Omega$ and $\psi_1 = 0 = \psi_2$ on $\Omega \setminus G$.

Note that

$$\begin{aligned} u_jT\wedge(dd^cu_j)^s - uT\wedge(dd^cu)^s &= (u_j-u)T\wedge(dd^cu_j)^s \\ &+ (v_1-w_1)(T\wedge(dd^cu_j)^s - T\wedge(dd^cu)^s) \\ &+ (\psi_1-\psi_2)(T\wedge(dd^cu_j)^s - T\wedge(dd^cu)^s). \end{aligned}$$

By the inductive hypothesis, we note that the second term

$$(v_1-w_1)(T\wedge(dd^cu_j)^s - T\wedge(dd^cu)^s) \longrightarrow 0 \quad\text{as}\quad j\to\infty.$$

Now we prove that $(u_j-u)T\wedge(dd^cu_j)^s \longrightarrow 0$ in weak* topology. Let $\varphi\in\mathcal{D}^{p-s,p-s}(\Omega)$, $\operatorname{supp}\varphi\Subset\Omega$. Choose $\Omega_1\Subset\Omega$ such that $\operatorname{supp}\varphi\subset\Omega_1$. Under the hypothesis ii), we can cover $\Omega_1\subset\bigcup\limits_{t=1}^m U_t$ with $U_t\Subset\Omega$ such that on each $U_t$ we can write $u_j=v_j^t-w_j^t, v_j^t, w_j^t\in \mathrm{PSH}(U_t)$ and the two sequences $\{v_j^t\},\{w_j^t\}$ are uniformly bounded on $U_t$ for all $j\geq 1$ and for all $1\leq t\leq m$. Then on each $U_t$ there is

$$T\wedge(dd^cu_j)^s\wedge\varphi = \sum_{r=0}^{s}(-1)^r\binom{s}{r}T\wedge(dd^cv_j^t)^r\wedge(dd^cw_j^t)^{s-r}\wedge\varphi.$$

Hence,

$$\begin{aligned} \Big|\int_\Omega (u_j-u)T\wedge(dd^cu_j)^s\wedge\varphi\Big| &\leq \int_\Omega |u_j-u|\|T\wedge(dd^cu_j)^s\wedge\varphi\| \\ &= \int_{\Omega_1}|u_j-u|\|T\wedge(dd^cu_j)^s\wedge\varphi\| \leq \sum_{t=1}^m\int_{U_t}|u_j-u|\|T\wedge(dd^cu_j)^s\wedge\varphi\| \\ &\leq \sum_{t=1}^m\sum_{r=0}^s\int_{U_t}\binom{s}{r}|u_j-u|\|T\wedge(dd^cv_j^t)^r\wedge(dd^cw_j^t)^{s-r}\wedge\varphi\| \\ &\leq \sum_{t=1}^m\sum_{r=0}^s \mathrm{B}_t\binom{s}{r}\int_{U_t}|u_j-u|T\wedge(dd^c(v_j^t+w_j^t+|z|^2))^p \qquad (3.4) \\ &= \sum_{t=1}^m\sum_{r=0}^s \mathrm{B}_t\binom{s}{r}\Big(\int\limits_{\{|u_j-u|>\delta\}\cap U_t} + \int\limits_{\{|u_j-u|\leq\delta\}\cap U_t}\Big), \end{aligned}$$

where $\mathrm{B}_t$ are some constants depending on $\varphi$ and inequality (3.4) holds because from Lemma 1.3.8 in [**2**] it follows that

$$\begin{aligned} \|T\wedge(dd^cv_j^t)^r\wedge(dd^cw_j^t)^{s-r}\varphi\| &\leq B_tT\wedge(dd^cv_j^t)^r\wedge(dd^cw_j^t)^{s-r}\wedge(dd^c|z|^2)^{p-s} \\ &\leq B_tT\wedge dd^c(v_j^t+w_j^t+|z|^2)^p. \end{aligned}$$

Let $\Omega_2 \Subset \Omega$ such that $\Omega_1 \subset \bigcup\limits_{t=1}^{m} U_t \subset \Omega_2 \Subset \Omega$. Then

$$\int\limits_{\{|u_j-u|\leq\delta\}\cap U_t} |u_j - u| T \wedge (dd^c(v_j^t + w_j^t + |z|^2))^p \leq \delta \mathrm{M}_t \|T\|_{\Omega_2},$$

where $\mathrm{M}_t$ is some constant independent of $j$ and $\|T\|_{\Omega_2}$ is the total variation of $T$ on $\Omega_2$. This estimate is obtained from Lemma 1.3.8 in [**2**] by using similar arguments as in the proof of Theorem 2.1.4 in [**2**]. Hence,

$$\sum_{t=1}^{m}\sum_{r=0}^{s} \mathrm{B}_t \binom{s}{r} \int\limits_{\{|u_j-u|\leq\delta\}\cap U_t} |u_j - u| T \wedge (dd^c(v_j^t + w_j^t + |z|^2))^p$$
$$\leq \delta \sum_{t=1}^{m}\sum_{r=0}^{s} \mathrm{B}_t \binom{s}{r} \mathrm{M}_t \|T\|_{\Omega_2} \leq \delta \mathrm{M} \|T\|_{\Omega_2}.$$

On the other hand,

$$\begin{aligned}
&\sum_{t=1}^{m}\sum_{r=0}^{s} \mathrm{B}_t \binom{s}{r} \int\limits_{\{|u_j-u|>\delta\}\cap U_t} |u_j - u| T \wedge (dd^c(v_j^t + w_j^t + |z|^2))^p \\
&\leq \sum_{t=1}^{m}\sum_{r=0}^{s} \mathrm{B}_t \binom{s}{r} \mathrm{A}_t \int\limits_{\{|u_j-u|>\delta\}\cap U_t} T \wedge (dd^c(v_j^t + w_j^t + |z|^2))^p \\
&\leq \sum_{t=1}^{m} \mathrm{N}_t \mathrm{C}_T\Big(\{\{|u_j - u| > \delta\} \cap U_t\}, U_t\Big) \longrightarrow 0
\end{aligned}$$

as $j \to \infty$, where $\mathrm{A}_t, \mathrm{N}_t$ do not depend on $j$. Thus $(u_j - u)T \wedge (dd^c u_j)^s \longrightarrow 0$ in weak* topology.

Theorem 3.1 will be proved if we show that $(\psi_1 - \psi_2)(T \wedge (dd^c u_j)^s - T \wedge (dd^c u)^s) \longrightarrow 0$ in weak* topology. Note that it is enough to prove that

$$\psi_1(T \wedge (dd^c u_j)^s - T \wedge (dd^c u)^s) \longrightarrow 0$$

in weak* topology. Let $\theta \in \mathcal{D}^{p-s,p-s}(\Omega)$ with $\operatorname{supp}\theta \Subset \Omega$. Since $\psi_1 = 0$ outside $G$, then we may assume that $\operatorname{supp}\theta \Subset G$. Choose $\operatorname{supp}\theta \Subset \Omega_3 \Subset G$. Then

$$\begin{aligned}
\Big|\int\limits_{\Omega}\psi_1 T\wedge(dd^cu_j)^s\wedge\theta\Big| &= \Big|\int\limits_{\Omega_3}\psi_1 T\wedge(dd^cu_j)^s\wedge\theta\Big|\\
&\le \int\limits_{\Omega_3}|\psi_1|\|T\wedge(dd^cu_j)^s\wedge\theta\| \le \mathrm{D}_1\int\limits_{\Omega_3}\|T\wedge(dd^cu_j)^s\wedge\theta\|\\
&\le \mathrm{D}_1\sum_{t=1}^{\ell}\int\limits_{U_t}\|T\wedge(dd^cu_j)^s\wedge\theta\|\\
&\le \mathrm{D}_1\sum_{t=1}^{\ell}\int\limits_{U_t}\sum_{r=0}^{s}\binom{s}{r}\|T\wedge(dd^cv_j^t)^r\wedge(dd^cw_j^t)^{s-r}\wedge\theta\| \qquad (3.5)\\
&\le \mathrm{D}_1\sum_{t=1}^{\ell}\sum_{r=0}^{s}\binom{s}{r}\mathrm{M}_t\int\limits_{U_t}T\wedge(dd^c(v_j^t+w_j^t+|z|^2))^p\\
&\le \mathrm{D}_1\sum_{t=1}^{\ell}\mathrm{H}_t\mathrm{C}_T(G,\Omega)\le\varepsilon\mathrm{D}_1\sum_{t=1}^{\ell}\mathrm{H}_t,
\end{aligned}$$

where $\operatorname{supp}\theta\subset\bigcup_{t=1}^{\ell}U_t\subset\Omega_3\Subset G$ and on each $U_t$ there is a representation $u_j=v_j^t-w_j^t, v_j^t, w_j^t\in\mathrm{PSH}(U_t)\cap\mathrm{L}^\infty(U_t)$, $\mathrm{D}_1$ and $\mathrm{H}_t$ are constants independent of $j$. Inequality (3.5) follows by using arguments similar to those used to prove inequality (3.4).

Similarly, one can prove that

$$\Big|\int\limits_{\Omega}\psi_1 T\wedge(dd^cu)^s\wedge\theta\Big|\le\varepsilon\mathrm{D}_1\mathrm{C}$$

and, therefore, the proof of Theorem 3.1 is finished. □

Now we establish the weak* convergence of $(dd^cu_j)^n$ to $(dd^cu)^n$ in the case of $u_j, u\in\delta\mathcal{E}_{loc}(\Omega)$. Namely, we prove the following.

THEOREM 3.2. *Let $\Omega$ be an open set in $\mathbb{C}^n$ and $u_j, u\in\delta\mathcal{E}_{loc}(\Omega)$. Assume that*

*i) $u_j\to u$ in $C_n$-capacity on every $E\Subset\Omega$.*

*ii) For each $z\in\Omega$ there exists a neighbourhood $U_z$ of $z$ in $\Omega$ such that on $U_z$ we can write*

$$u_j=v_j-w_j,\quad u=v-w,$$

*where $v_j, w_j, v, w\in\mathcal{E}(U_z)$ and $|v_j|\le|g|, |w_j|\le|g|, |v|\le|g|, |w|\le|g|$ on $U_z$, $g\in\mathcal{E}(\Omega)$.*

*Then $(dd^cu_j)^n$ is weak* convergence to $(dd^cu)^n$ in $\Omega$.*

In order to prove the above theorem, we need the following

LEMMA 3.3. *Let $\Omega \subset \mathbb{C}^n$ be a hyperconvex domain and $E \subset \Omega$ a Borel subset. Assume that $\varphi_1, \cdots, \varphi_{n-1} \in PSH^-(\Omega), g \in \mathcal{F}(\Omega)$ are such that $\varphi_i \geq g, i = 1, 2, \cdots, n-1$. Then for every $\varphi \in \mathcal{E}_0(\Omega)$, we have*

$$\int_E dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} \leq \|\varphi\|_{L^\infty(\Omega)} \Big[C_n(E)\Big]^{\frac{1}{n}} \Big[\int_\Omega (dd^c g)^n\Big]^{\frac{n-1}{n}}, \tag{3.6}$$

*where $PSH^-(\Omega)$ denotes the set of negative plurisubharmonic functions on $\Omega$.*

PROOF. First we assume that $E$ is a relatively compact open set in $\Omega$. Let $h^*_{E,\Omega}$ denote the upper semicontinuous regularization of the relatively extremal function $h_{E,\Omega}$ of $E$. Then $h^*_{E,\Omega} \in \mathcal{E}_0(\Omega)$. Moreover, $h^*_{E,\Omega} \equiv -1$ on $E$ and $-1 \leq h^*_{E,\Omega} \leq 0$ on $\Omega$. It follows that

$$\begin{aligned}
&\int_E dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} \\
&\qquad \leq \int_\Omega -h^*_{E,\Omega} dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} \\
&\qquad = \int_\Omega -\varphi dd^c h^*_{E,\Omega} \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} \\
&\qquad \leq \Big[\int_\Omega -\varphi (dd^c h^*_{E,\Omega})^n\Big]^{\frac{1}{n}} \prod_{j=1}^{n-1} \Big[\int_\Omega -\varphi (dd^c\varphi_j)^n\Big]^{\frac{1}{n}} \\
&\qquad \leq \|\varphi\|_{L^\infty(\Omega)} \Big[\int_\Omega (dd^c h^*_{E,\Omega})^n\Big]^{\frac{1}{n}} \prod_{j=1}^{n-1} \Big[\int_\Omega (dd^c\varphi_j)^n\Big]^{\frac{1}{n}},
\end{aligned} \tag{3.7}$$

where the inequality in the fourth line follows from Theorem 5.5 in [**4**].

On the other hand, since $\varphi_j \geq g, j = 1, \ldots, n-1$, then $\varphi_j \in \mathcal{F}(\Omega)$ and by using the Remark after Definition 4.6 together with Theorem 3.2 in [**4**], we obtain

$$\int_\Omega (dd^c\varphi_j)^n \leq \int_\Omega (dd^c g)^n. \tag{3.8}$$

Moreover, Proposition 4.6.1 in [**10**] implies that

$$\int_\Omega (dd^c h^*_{E,\Omega})^n = C_n(E). \tag{3.9}$$

Combining (3.7), (3.8) and (3.9), we get desired inequality (3.6) in the case of $E$ a relatively compact open set in $\Omega$.

Now suppose that $E$ is a compact subset of $\Omega$. Take a decreasing sequence $\{D_k\}_{k=1}^{\infty}$ of relatively compact open sets of $\Omega$ such that $D_k \searrow E$ as $k \to \infty$. Then

$$\lim_{k\to\infty} \int_{D_k} dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} = \int_E dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1}$$

and

$$\lim_{k\to\infty} C_n(D_k) = C_n(E).$$

Applying inequality (3.6) to $D_k$ and passing with $k$ to $\infty$, we obtain the desired conclusion.

Finally, assume that $E \subset \Omega$ is a Borel subset. Let $\{K_m\}_{m\geq 1}$ be an increasing sequence of compact subsets such that $K_m \subset E$ and $K_m \nearrow E$. Then

$$C_n(E) = \lim_{m\to\infty} C_n(K_m)$$

and

$$\lim_{m\to\infty} \int_{K_m} dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1} = \int_E dd^c\varphi \wedge dd^c\varphi_1 \wedge \cdots \wedge dd^c\varphi_{n-1}$$

and by the result of the second case we get inequality (3.6). The proof of the lemma is complete. $\square$

Proof of Theorem 3.2. Since the problem is local, then we may assume that for each $z \in \Omega$ there exists a neighbourhood $U_z$ of $z$ in $\Omega$ as in the statement of the theorem and it suffices to prove that $(dd^c u_j)^n$ is weak* convergent to $(dd^c u)^n$ on $U_z$. Since the class $\mathcal{E}$ has the local property (see the proof of Theorem 1.1 in [**3**]), then we may assume that $g \in \mathcal{E}(U_z)$. Thus in the proof of the theorem we may assume that $U = U_z$ is a hyperconvex domain in $\mathbb{C}^n$, $u_j = v_j - w_j$, $v_j, w_j \in \mathcal{E}(U)$ and we have to prove that

$$(dd^c u_j)^n \longrightarrow (dd^c u)^n \ \text{ in weak* topology in } \ U.$$

Take $\varphi \in \mathrm{C}_0^{\infty}(U)$. We may assume that $-1 \leq \varphi \leq 0$. Write

$$\begin{aligned}
\Big|\int_U \varphi(dd^c u_j)^n - \int_U \varphi(dd^c u)^n\Big| &= \Big|\int_U \varphi((dd^c u_j)^n - (dd^c u)^n)\Big| \\
&= \Big|\int_U \varphi dd^c(u_j - u) \wedge \Big(\sum_{k=0}^{n-1} (dd^c u_j)^k \wedge (dd^c u)^{n-1-k}\Big)\Big| \qquad (3.10)\\
&= \Big|\int_U (u_j - u) dd^c\varphi \wedge \Big(\sum_{k=0}^{n-1} (dd^c u_j)^k \wedge (dd^c u)^{n-1-k}\Big)\Big|.
\end{aligned}$$

By replacing $u_j = v_j - w_j, u = v - w$ in (3.10), we get the following estimate

$$\begin{aligned}
&\Big|\int\limits_U \varphi(dd^c u_j)^n - \int\limits_U \varphi(dd^c u)^n\Big| \\
&\quad \leq \sum_{\text{finite}} \Big|\int\limits_U (u_j - u) dd^c\varphi \wedge dd^c\psi_{j_2} \wedge \cdots \wedge dd^c\psi_{j_{n-1}}\Big|,
\end{aligned}$$

where either $\psi_{j_k} = v_k, w_k$ or $\psi_{j_k} = v, w$. Moreover, under the hypothesis it follows that $\psi_{j_k} \geq g$ on $U$ for all $k = 2, \cdots, n-1$. Hence, it remains to show that

$$\lim_{j\to\infty}\Big|\int\limits_U (u_j - u) dd^c\varphi \wedge dd^c\psi_{j_2} \wedge \cdots \wedge dd^c\psi_{j_{n-1}}\Big| = 0$$

for $\psi_{j_k} \geq g$ on $U$ for $k = 2, 3, \cdots, n-1$. Take an open set $\mathbb{D} \Subset U$ such that $\operatorname{supp}\varphi = K \Subset \mathbb{D}$. Then

$$\begin{aligned}
&\Big|\int\limits_U (u_j - u) dd^c\varphi \wedge dd^c\psi_{j_2} \wedge \cdots \wedge dd^c\psi_{j_{n-1}}\Big| \\
&\quad = \Big|\int\limits_{\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\psi_{j_2} \wedge \cdots \wedge dd^c\psi_{j_{n-1}}\Big|.
\end{aligned} \tag{3.11}$$

Now, by [**4**], there exists $\widetilde{g} \in \mathcal{F}(U)$ such that $\widetilde{g} = g$ on $\mathbb{D}$. Put $\widetilde{\psi}_{j_k} = \max(\psi_{j_k}, \widetilde{g}) \in \mathcal{F}(U), \widetilde{\psi}_{j_k} = \psi_{j_k}$ on $\mathbb{D}$. Let $\varepsilon > 0$ be given. Then we can write (3.11) as follows.

$$\begin{aligned}
(3.11) = &\Big|\int\limits_{\{|u_j - u|\leq\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \\
&+ \int\limits_{\{|u_j - u|>\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}}\Big| \\
\leq &\Big|\int\limits_{\{|u_j - u|\leq\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}}\Big| \\
&+ \Big|\int\limits_{\{|u_j - u|>\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}}\Big|.
\end{aligned}$$

Set

$$A_j = \Big| \int\limits_{\{|u_j-u|\le\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \Big|$$

and

$$B_j = \Big| \int\limits_{\{|u_j-u|>\varepsilon\}\cap\mathbb{D}} (u_j - u) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \Big|.$$

Hence, the proof of the theorem is complete if we prove that

$$\lim_{j\to\infty} A_j = 0$$

and

$$\lim_{j\to\infty} B_j = 0.$$

By Lemma 3.1 in [**4**], we can write $\varphi = \varphi_1 - \varphi_2$, where $\varphi_1, \varphi_2 \in \mathcal{E}_0$. So we can assume that $\varphi \in \mathcal{E}_0$. Since $\widetilde{\psi}_{j_k} \in \mathcal{F}(U), \widetilde{g} \in \mathcal{F}(U)$ and $\widetilde{\psi}_{j_k} \ge \widetilde{g}$ on $U$, then by the remark after Definition 4.6 in [**4**], there holds

$$\begin{aligned} A_j &\le \int\limits_{\{|u_j-u|\le\varepsilon\}\cap\mathbb{D}} |(u_j - u)| dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \\ &\le \varepsilon \int\limits_{\mathbb{D}} dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \\ &\le \varepsilon \int\limits_{U} dd^c\varphi \wedge (dd^c\widetilde{g})^{n-1}. \end{aligned}$$

Let $X_j = \{|u_j - u| > \varepsilon\} \cap \mathbb{D}$. Then

$$B_j \le \int\limits_{X_j} (|u_j| + |u|) dd^c\varphi \wedge dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}} \le 4 \int\limits_{X_j} (-\widetilde{g}) dd^c\varphi \wedge \widetilde{T}_2,$$

where $\widetilde{T}_2 = dd^c\widetilde{\psi}_{j_2} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}}$.

Now for each $R > 0$, put $\widetilde{g}_R = \max(\widetilde{g}, -R)$. Then

$$\begin{aligned} B_j &\le 4 \int\limits_{X_j} (-\widetilde{g} + \widetilde{g}_{2^n R}) dd^c\varphi \wedge \widetilde{T}_2 + 4 \int\limits_{X_j} (-\widetilde{g}_{2^n R}) dd^c\varphi \wedge \widetilde{T}_2 \\ &\le 4 \int\limits_{X_j} (-\widetilde{g} + \widetilde{g}_{2^n R}) dd^c\varphi \wedge \widetilde{T}_2 + 2^{n+2} R \int\limits_{X_j} dd^c\varphi \wedge \widetilde{T}_2, \end{aligned}$$

because $-\widetilde{g}_{2^n R} \le 2^n R$. Lemma 3.3 implies that

$$\int\limits_{X_j} dd^c\varphi \wedge \widetilde{T}_2 \le \Big[C_n(\{|u_j - u| > \varepsilon\}, U)\Big]^{\frac{1}{n}} \Big[\int\limits_U (dd^c\widetilde{g})^n\Big]^{\frac{n-1}{n}}$$

and, hence, by the hypothesis, we get

$$\lim_{j\to\infty} \int\limits_{X_j} dd^c\varphi \wedge \widetilde{T}_2 = 0.$$

Thus, it follows that

$$\limsup_{j\to\infty} B_j \le 4 \sup_{j\ge 1} \int\limits_U (-\widetilde{g} + \widetilde{g}_{2^n R}) dd^c\varphi \wedge \widetilde{T}_2. \tag{3.12}$$

We give the estimate of the right hand side of (3.12) as follows.

$$\begin{aligned}
&\int\limits_U (-\widetilde{g} + \widetilde{g}_{2^n R}) dd^c\varphi \wedge \widetilde{T}_2 = \int\limits_U -\widetilde{\psi}_{j_2} dd^c\varphi \wedge dd^c(\widetilde{g} - \widetilde{g}_{2^n R}) \wedge \widetilde{T}_3 \\
&= \int\limits_{\{\widetilde{g} \le -2^n R\}} -\widetilde{\psi}_{j_2} dd^c\varphi \wedge dd^c(\widetilde{g} - \widetilde{g}_{2^n R}) \wedge \widetilde{T}_3 \\
&= \int\limits_{\{\widetilde{g} \le -2^n R\}} -\widetilde{\psi}_{j_2} dd^c\varphi \wedge dd^c\widetilde{g} \wedge \widetilde{T}_3 - \int\limits_{\{\widetilde{g} \le -2^n R\}} -\widetilde{\psi}_{j_2} dd^c\varphi \wedge dd^c\widetilde{g}_{2^n R} \wedge \widetilde{T}_3 \\
&\le \int\limits_{\{\widetilde{g} \le -2^n R\}} -\widetilde{\psi}_{j_2} dd^c\varphi \wedge dd^c\widetilde{g} \wedge \widetilde{T}_3 \le \int\limits_{\{\widetilde{g} \le -2^n R\}} -\widetilde{g} dd^c\varphi \wedge dd^c\widetilde{g} \wedge \widetilde{T}_3 \\
&\le 2 \int\limits_{\{\widetilde{g} \le -2^n R\}} (-\widetilde{g} + \widetilde{g}_{2^{n-1} R}) dd^c\varphi \wedge dd^c\widetilde{g} \wedge \widetilde{T}_3 \\
&\le 2 \int\limits_{\{\widetilde{g} \le -2^{n-1} R\}} (-\widetilde{g} + \widetilde{g}_{2^{n-1} R}) dd^c\varphi \wedge dd^c\widetilde{g} \wedge \widetilde{T}_3,
\end{aligned}$$

because $-\widetilde{g} \le 2(-\widetilde{g} + \widetilde{g}_{2^{n-1}R})$ on the set $\{\widetilde{g} \le -2^n R\}$ and $\{\widetilde{g} \le -2^n R\} \subset \{\widetilde{g} \le -2^{n-1} R\}$ and $\widetilde{T}_3 = dd^c\widetilde{\psi}_{j_3} \wedge \cdots \wedge dd^c\widetilde{\psi}_{j_{n-1}}$.

By repeating the same arguments $n$ times, we arrive at

$$\limsup_{j\to\infty} B_j \le 2^n \int\limits_{\{\widetilde{g} \le -R\}} -\widetilde{g} dd^c\varphi \wedge (dd^c\widetilde{g})^{n-1}. \tag{3.13}$$

However,

$$\int\limits_{\{\widetilde{g}\le -R\}} -\widetilde{g}dd^c\varphi\wedge(dd^c\widetilde{g})^{n-1}\le\int\limits_U -\widetilde{g}dd^c\varphi\wedge(dd^c\widetilde{g})^{n-1}$$

$$\le\int\limits_U -\varphi(dd^c\widetilde{g})^n\le\int\limits_U (dd^c\widetilde{g})^n<\infty.$$

Hence, $-\widetilde{g}\in \mathrm{L}^1(dd^c\varphi\wedge(dd^c\widetilde{g})^{n-1})$. On the other hand, since $\varphi\in\mathcal{E}_0(U)$ then Theorem 2.1 in [**4**] and Lemma 3.3 imply that $dd^c\varphi\wedge(dd^c\widetilde{g})^{n-1}$ is absolutely continuous with respect to $C_n$-capacity. By the Radon–Nikodym theorem, $-\widetilde{g}dd^c\varphi\wedge(dd^c\widetilde{g})^{n-1}$ is also absolutely continuous with respect to $C_n$-capacity. But Proposition 3.1 in [**7**] implies that

$$C_n(\{\widetilde{g}\le -R\},U)\le\frac{M_n\int\limits_U(dd^c\widetilde{g})^n}{R^n}\longrightarrow 0$$

as $R\to\infty$, where $M_n$ is a constant.

If $R\to\infty$ in the right-hand side of (3.13), we infer that $\limsup\limits_{j\to\infty} B_j=0$. Hence, the proof of Theorem 3.2 is complete. □

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