ON THE STABLE EQUIVALENCE PROBLEM

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Abstract

In this paper we deal with the stable equivalence problem in a "strong version" over an algebraically closed field $\mathbb{K}$ of characteristic 0. For this purpose we introduce a notion of stabilizing hypersurfaces, i.e., a hypersurface H in an affine variety X is called *stabilizing* if every isomorphism $f : X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ such that $f(H \times \mathbb{K}^m) = H' \times \mathbb{K}^m$, where H' is a hypersurface in a variety Y, satisfies the condition: for each $x \in X$ there exists $y \in Y$ such that $f(\{x\} \times \mathbb{K}^m) = \{y\} \times \mathbb{K}^m$ (obviously, then f induces the isomorphism $\tilde{f} : X \to Y$ such that $\tilde{f}(H) = H'$). We prove that a non-uniruled hypersurface in a smooth affine variety and a non-$\mathbb{C}$-uniruled hypersurface in a smooth affine and dominated by $\mathbb{C}^n$ variety are examples of stabilizing hypersurfaces. In particular, the stable equivalence problem has an affirmative solution for any non-$\mathbb{C}$-uniruled hypersurface in $\mathbb{C}^n$.

1 Introduction

Throughout this paper $\mathbb{K}$ denotes an algebraically closed field of characteristic 0. Let X and Y be algebraic sets in $\mathbb{K}^n$. Following [17], we say that X and Y are *equivalent* if there exists an automorphism of $\mathbb{K}^n$ that takes X onto Y. Furthermore, we say that X and Y are *stably equivalent* if for some $m \geq 0$ the cylinders $X \times \mathbb{K}^m$ and $Y \times \mathbb{K}^m$ are equivalent in $\mathbb{K}^{n+m}$.

STABLE EQUIVALENCE PROBLEM. Are any two stably equivalent hypersurfaces in $\mathbb{K}^n$ equivalent?

This problem for curves in $\mathbb{K}^2$ was solved affirmatively by Makar-Limanov, van Rossum, Shpilrain and Yu in [17]. Furthermore, it was proved by Shpilrain and Yu in [18] that two stably equivalent hypersurfaces in $\mathbb{C}^n$ are equivalent if one of them is the set of zeros of a so-called test polynomial in the class of monomorphisms. The same was proved in [3] provided that one hypersurface is non-uniruled.

In this paper we will generalize the last result. We will consider the stable equivalence problem in a stronger version. For this purpose we introduce a notion of stabilizing hypersurfaces. (Let us note that we will mean by a variety an irreducible algebraic variety, and by a hypersurface in a variety a closed subset of pure codimension one.)

A hypersurface H in an affine variety X is called *stabilizing* if it satisfies the condition: if $f : X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ is an isomorphism such that $f(H \times \mathbb{K}^m) = H' \times \mathbb{K}^m$, where H' is a hypersurface in a variety Y, then for each $x \in X$ there exists $y \in Y$ such that $f(\{x\} \times \mathbb{K}^m) = \{y\} \times \mathbb{K}^m$ (obviously, then f induces the isomorphism between X and Y that takes H onto H').

We will give some sufficient geometric condition for a hypesurface to be stabilizing, namely, a non-uniruled hypersurface in a smooth affine variety and a non-$\mathbb{C}$-uniruled hypersurface in a smooth affine and dominated by $\mathbb{C}^n$ variety are stabilizing. In particular, the stable equivalence problem has an affirmative solution for non-$\mathbb{C}$-uniruled hypersurfaces in $\mathbb{C}^n$.

This article is divided into five section. In section 2 we give some examples of uniruled varieties. Section 3 contains a brief summary of properties of the Jelonek set. In sections 4 and 5 are stated and proved our main results mentioned above. Furthermore, we prove there that the set of zeros of a test polynomial in the class of monomorphisms is a stabilizing hypersurface in $\mathbb{C}^n$ (this is a sharpened version of the result from [18]).

2 Uniruledness

A variety X of dimension $n > 0$ is said to be *uniruled* ($\mathbb{K}$-*uniruled*) if there exists a variety W of dimension $n-1$ and a dominant rational map $W \times \mathbb{P}^1(\mathbb{K}) \dashrightarrow X$ (a dominant morphism $W \times \mathbb{K} \to X$). A reducible variety is said to be *uniruled* ($\mathbb{K}$-*uniruled*) if all its irreducible components are *uniruled* ($\mathbb{K}$-*uniruled*).

Example 2.1. *(i) Let X be an affine variety of dimension n. Suppose that $F_1, \dots, F_n$ are algebraically independent regular functions on X. Then the variety $V = X \setminus \{x \in X : F_1(x) \dots F_n(x) = 0\}$ is non-$\mathbb{K}$-uniruled.*
(ii) A smooth hypersurface in $\mathbb{P}^n$ of degree greater than n is non-uniruled.

Proof. (i) Since we have a dominant morphism $(F_1, \dots, F_n) : V \to (\mathbb{K}^*)^n$, it suffices to show that $(\mathbb{K}^*)^n$ is non-$\mathbb{K}$-uniruled. Let $g : W \times \mathbb{K} \to (\mathbb{K}^*)^n$ be a morphism, where W is affine of dimension $n-1$. If $g = (G_1, \dots, G_n)$ then all function G_i are invertible in the coordinate ring $\mathbb{K}[W \times \mathbb{K}]$, and hence they are in $\mathbb{K}[W]$. Thus g is not dominant.
(ii) see [12]. □

We say that a variety X has the *strong cancellation property* if every isomorphism $f : X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ satisfies the condition: for each $x \in Y$ there exists $y \in X$ such that $f(\{x\} \times \mathbb{K}^m) = \{y\} \times \mathbb{K}^m$ (clearly, f induces the isomorphism between X and Y). For example, a variety of non-negative logarithmic Kodaira dimension has this property, which was proved by Iitaka and Fujita in [10]. Other examples are given by

Proposition 2.2. ([4]) *A non-$\mathbb{K}$-uniruled affine variety has the strong cancellation property.*

3 The Jelonek set

Let $f : X \to Y$ be a morphism between varieties. We say that f is *proper* at $y \in Y$ if there exists a neighborhood U of y such that $\operatorname{res} f : f^{-1}(U) \to U$ is a proper morphism. It is well-known that over $\mathbb{C}$ a morphism f is proper at y if and only if f is proper at y in the $\mathbb{C}$-topology, i.e., there exists an open (in the $\mathbb{C}$-topology) neighborhood U of y such that for each compact set $T \subset U$ the inverse image $f^{-1}(T)$ is compact.

A very important role in our consideration will play the *Jelonek set*

$$S_f := \{y \in Y : f \text{ is not proper at } y\}.$$

Theorem 3.1. (Jelonek, [13].) *Let $f : X \to Y$ be a dominant morphism between complex affine varieties of equal dimension. Suppose that X is dominated by $\mathbb{C}^n$. Then the set S_f is either empty or it is a $\mathbb{C}$-uniruled hypersurface.*

The first version of this theorem was stated in [11], where the author proved that for a dominant morphism $f : \mathbb{C}^n \to \mathbb{C}^n$ the set S_f is either empty or it is a uniruled hypersurface. We will use the same idea to establish a little more general result below, which will be needed in the proof of Theorem 4.3. Note, that Theorem 3.1 was partially carried over an arbitrary algebraically closed field by Stasica [19]. Other generalization was given by Jelonek and Karaś in [15].

Now we give two well-known facts.

(1) By theorem of Hironaka [8] for a smooth affine variety there exists a smooth compactification (we mean by a *compactification* of a variety X any projective variety which contains X as an open subset).

(2) If X is an affine variety and a variety X' contains X as an open subset then the set $X' \setminus X$ is either empty or it is of pure dimension $\dim X - 1$.

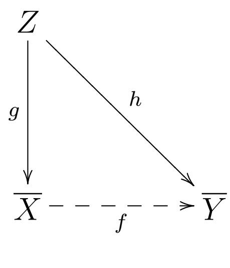
Theorem 3.2. *Let $f : X \to Y$ be a dominant morphism between affine varieties of dimension n. Suppose that X is smooth. Let $\overline{X}$ be a smooth compactification of X and $\overline{Y}$ a compactification of Y. Let*

$r :=$ number of non-uniruled irreducible components of the set $\overline{X} \setminus X$,
$s :=$ number of non-uniruled irreducible components of the set $\overline{Y} \setminus Y$.

Then
(i) $r \geq s$;
(ii) if $r = s$ then every $(n-1)$-dimensional irreducible component of the set S_f is uniruled.

In particular, if X and Y are smooth affine varieties of dimension n such that X dominates Y and conversely, and if $f : X \to Y$ is a dominant morphism, then every $(n-1)$-dimensional irreducible component of the set S_f is uniruled.

Proof. The idea of this proof is due to [11,12]. By [8] for the rational map $f : \overline{X} \dashrightarrow \overline{Y}$ there exists a commutative diagram



where $g : Z \to \overline{X}$ is a composition of blowing ups with smooth centers and h is a morphism. Let $E \subset Z$ be the largest exceptional divisor for g. By properties of blowing up E is ruled, i.e., is birational to $V \times \mathbb{P}^1$. Consequently every $(n-1)$-dimensional irreducible component of the set $h(E)$ is uniruled. Clearly, $\overline{Y} \setminus Y$ is contained in $h(g^{-1}(\overline{X} \setminus X)) = h((\overline{X} \setminus X)') \cup h(E)$, where $(\overline{X} \setminus X)'$ denotes the proper transform of $\overline{X} \setminus X$ by g. Therefore every non-uniruled component of $\overline{Y} \setminus Y$ coincides with some component of $h((\overline{X} \setminus X)')$. This proves (i). To prove (ii) it suffices to check that $S_f \subset h(g^{-1}(\overline{X} \setminus X))$. This is equivalent that $\operatorname{res} f : f^{-1}(U) \to U$ is a proper morphism, where $U = \overline{Y} \setminus h(g^{-1}(\overline{X} \setminus X))$. It is clear that $f^{-1}(U)$ is contained in X and the following diagram commutes

$$\begin{array}{ccc} g^{-1}(f^{-1}(U)) = h^{-1}(U) & & \\ \quad\downarrow \operatorname{res} g & \searrow^{\operatorname{res} h} & \\ f^{-1}(U) & \xrightarrow[\operatorname{res} f]{} & U \end{array}$$

Since the vertical arrow is surjective and the wedge arrow is proper, the horizontal arrow is proper too. This completes the proof. □

4 Stabilizing hypersurfaces

We shall begin with an easy necessary condition for a hypersurface to be stabilizing.

Proposition 4.1. *Let X be an affine variety. Suppose that F is a regular function on X such that the set $F^{-1}(0)$ is a stabilizing hypersurface. Then $D(F) \neq 0$ for every non-zero locally nilpotent $\mathbb{K}$-derivation D on the coordinate ring $\mathbb{K}[X]$.*

Proof. A locally nilpotent $\mathbb{K}$-derivation D on $\mathbb{K}[X]$ induces the automorphism $\exp TD : \mathbb{K}[X][T] \ni G \mapsto \sum_{i=0}^{\infty} \frac{1}{i!} D^i(G) T^i \in \mathbb{K}[X][T]$, where D is extended on $\mathbb{K}[X][T]$ such that $D(T) = 0$ (see [5, p.17]). Therefore if $D(F) = 0$ and a hypersurface $F^{-1}(0)$ is stabilizing then $\exp TD(F) = F$, and consequently $\exp TD(\mathbb{K}[X]) \subset \mathbb{K}[X]$, so $D = 0$. □

Note the following

Proposition 4.2. *Let H be a hypersurface in an affine variety X. Suppose that the variety $X \setminus H$ has the strong cancellation property (in particular, if $X \setminus H$ is either non-$\mathbb{K}$-uniruled or if $\overline{\kappa}(X \setminus H) \geq 0$). Then H is stabilizing.*

For example, if $n = \dim X$ and $F_1, \dots, F_n$ are algebraically independent regular functions on X then $\{x \in X : F_1(x) \dots F_n(x) = 0\}$ is a stabilizing hypersurface.

Proof. The assertion follows immediately from section 2 and an easy observation that for a morphism $f : Y \times \mathbb{K}^m \to Z$ the set $\{y \in Y : \dim f(\{y\} \times \mathbb{K}^m) = 0\}$ is closed in Y. $\square$

Now we give the main result of this section.

Theorem 4.3. *A non-uniruled hypersurface contained in a smooth affine variety is stabilizing.*

We will need three lemmas.

Lemma 4.4. *Let $f : X \to Y$ be a dominant morphism between smooth varieties of equal dimension. Suppose that f is étale at $a \in X$, proper at $b = f(a)$, and $f^{-1}(b) = \{a\}$. Then there exists a neighborhood U of b such that* $\operatorname{res} f : f^{-1}(U) \to U$ *is an isomorphism.*

Proof. We may assume that f is proper. Let R_f be the set of points at which f is not étale. Then $f(R_f)$ is closed in Y and $b \in Y \setminus f(R_f)$. Replacing $Y \setminus f(R_f)$ by Y and $X \setminus f^{-1}(f(R_f))$ by X we may assume that f is étale. Hence f is an étale covering, so it is a finite morphism (see [9, p.248]). Now we may assume that X and Y are affine. Consider the corresponding finite extension of coordinate rings $\mathbb{K}[Y] \subset \mathbb{K}[X]$. Let $(\vartheta_x, \mathfrak{m}_x)$ and $(\vartheta_y, \mathfrak{m}_y)$ be local rings of points x and y, respectively. Put $M = \mathfrak{m}_y \cap \mathbb{K}[Y]$. Clearly, the extension $\vartheta_y = \mathbb{K}[Y]_M \subset \mathbb{K}[X]_{\mathbb{K}[Y] \setminus M}$ is finite and the ring $\mathbb{K}[X]_{\mathbb{K}[Y] \setminus M}$ has a unique maximal ideal. Thus $\mathbb{K}[X]_{\mathbb{K}[Y] \setminus M} = \vartheta_x$ and consequently ϑ_x is a finitely generated ϑ_y-module. Moreover, $\mathfrak{m}_x = \mathfrak{m}_y \vartheta_x$, since f is étale at x. Because $\vartheta_x / \mathfrak{m}_x \cong \vartheta_y / \mathfrak{m}_y \cong \mathbb{K}$, so $\vartheta_x = \vartheta_y + \mathfrak{m}_x = \vartheta_y + \mathfrak{m}_y \vartheta_x$. Hence, by Nakayama's lemma, $\vartheta_y = \vartheta_x$. From this the assertion follows at once.

Note that for complex varieties we may argue as follows. Choose an open (in the $\mathbb{C}$-topology) neighborhood V of a such that $f : V \to f(V)$ is a biholomorphism. By properness of f at b there is an open neighborhood U of b such that $f^{-1}(U)$ is contained in V. This means that the generic fiber of f consists of one point, so f is birational. By Zariski's Main Theorem f^{-1} is regular at b. $\square$

In the sequel π_X denotes the projection $X \times \mathbb{K}^m \ni (x, y) \mapsto x \in X$.

Lemma 4.5. *Let X be a smooth affine variety and W be a subvariety of $X \times \mathbb{K}^m$. Suppose that W is non-singular at points $a_1, \dots, a_s \in W$ and $\pi_X(a_i) \neq \pi_X(a_j)$ for $i \neq j$. Then there exists a morphism $p : X \to \mathbb{K}^m$ such that its graph meets W transversally at $a_1, \dots, a_s$.*

Proof. It suffices to give a proof for $X = \mathbb{K}^n$. (Indeed, if X is contained in $\mathbb{K}^n$ and a morphism $p : \mathbb{K}^n \to \mathbb{K}^m$ satisfies the above condition then $\operatorname{res}_X p$ satisfies this condition too.) Choose n-dimensional affine subspaces $L_1, \ldots, L_s$ in $\mathbb{K}^n \times \mathbb{K}^m$ such that L_i and W are transversal at a_i and the projection $L_i \to \mathbb{K}^n$ is an isomorphism for all i. Using an easy interpolation we can find a polynomial mapping $p : \mathbb{K}^n \to \mathbb{K}^m$ the graph of which pass through $a_1, \ldots, a_s$ and has L_i as a tangent space at a_i for all i. This concludes the proof. □

Lemma 4.6. *Let $f : X \to Y$ be a birational morphism between smooth varieties. If a fiber of f contains at least two points then its all irreducible components have positive dimension.*

Proof. Let $f^{-1}(y)$ satisfy the assumptions. Since f^{-1} is not regular at y, by Zariski's Main Theorem there exists an exceptional divisor E containing $f^{-1}(y)$. Hence all irreducible components of $f^{-1}(y) = (f|_E)^{-1}(y)$ have positive dimension. □

Proof of Theorem 4.3. Let X be a smooth affine variety and H a non-uniruled hypersurface in X. Let $f : X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ be an isomorphism such that $f(H \times \mathbb{K}^m) = H' \times \mathbb{K}^m$, where H' is a hypersurface in Y. We may assume that H is irreducible.

For any morphism $p : X \to \mathbb{K}^m$ define the morphism $f_p : X \ni x \mapsto \pi_Y(f(x, p(x))) \in Y$. We shall show that f_p is birational.

Let $a \in X$, $A := (a, p(a))$ and Γ_p denote the graph of p. By Proposition 2.2, $f(\{a\} \times \mathbb{K}^m) = \{b\} \times \mathbb{K}^m$ for some $b \in H'$. Clearly, Γ_p and $\{a\} \times \mathbb{K}^m$ are transversal at A, hence $f(\Gamma_p)$ and $\{b\} \times \mathbb{K}^m$ are transversal at $f(A)$ too. Consequently $\operatorname{res} \pi_Y : f(\Gamma_p) \to Y$ is étale at $f(A)$. This implies that f_p is étale at a as a composition of étale morphisms. It is clear that $f_p : H \to H'$ is an isomorphism and $f_p^{-1}(f_p(x)) = \{x\}$ for each $x \in H'$. Furthermore, X dominates Y and conversely, so by Theorem 3.2, f_p is proper at some point of H'. It implies that f_p is birational by Lemma 4.4.

Suppose now that $\pi_X(f^{-1}(\{y\} \times \mathbb{K}^m))$ has positive dimension for some $y \in Y$. Choose two points $a, b \in f^{-1}(\{y\} \times \mathbb{K}^m)$ such that $\pi_X(a) \neq \pi_X(b)$. By Lemma 4.5 there exists a morphism $p : X \to \mathbb{K}^m$ such that its graph meets transversally $f^{-1}(\{y\} \times \mathbb{K}^m)$ at a, b. Hence $\{\pi_X(a)\}$ and $\{\pi_X(b)\}$ are components of the fiber $f_p^{-1}(y)$, which is impossible by Lemma 4.6. The proof is complete. □

5 Stabilizing hypersurfaces over $\mathbb{C}$

Theorem 5.1. *Let X be a complex affine and smooth variety which is dominated by $\mathbb{C}^n$. Then any non-$\mathbb{C}$-uniruled hypersurface in X is stabilizing.*

Proof. The proof of Theorem 4.3 works also in this case, we need only use Theorem 3.1 instead of Theorem 3.2. □

Corollary 5.2. *Two stably equivalent hypersurfaces in $\mathbb{C}^n$ are equivalent if one of them is non-$\mathbb{C}$-uniruled.*

Corollary 5.3. *Let X be an irreducible curve in $\mathbb{C}^2$ which is either non-$\mathbb{C}$-uniruled or it is simply connected. Then any curve in $\mathbb{C}^2$ stably equivalent to X is equivalent to it.*

Proof. It remains to prove the case when X is simply connected, but this follows at once from two remarks. (1) By theorems of Abhyankar-Moh [2] and Lin-Zaidenberg [16] (see also [7]) we know that two simply connected curves in $\mathbb{C}^2$ are isomorphic if and only if they are equivalent. (2) Affine curves have the cancellation property, i.e., if C_1 and C_2 are affine curves and $C_1 \times \mathbb{K}^m \cong C_2 \times \mathbb{K}^m$ then $C_1 \cong C_2$ (this follows, for example, from a more general theorem proved by Abhyankar, Eakin and Heinzer in [1]). □

Our next purpose is to give a sharpened version of the result of Shpilrain and Yu from [18]. Recall that a polynomial $P \in \mathbb{C}[T_1, \dots, T_n]$ is said to be *a test polynomial in the class of monomorphisms* if every $\mathbb{C}$-monomorphism $\Phi : \mathbb{C}[T_1, \dots, T_n] \to \mathbb{C}[T_1, \dots, T_n]$ such that $\Phi(P) = cP$ for some $c \in \mathbb{C}^*$ is an automorphism. A question about the existence and some properties of such polynomials were stated by van den Essen and Shpilrain in [6]. Later it was showed by Jelonek in [14] that a generic polynomial in $\mathbb{C}[T_1, \dots, T_n]$ of degree greater than n is a test polynomial in the class of monomorphisms. (Note that the above definition is due to [14] and it is slightly different than that introduced in [6].)

Proposition 5.4. *Let $P \in \mathbb{C}[T_1, \dots, T_n]$ be a test polynomial in the class of monomorphisms. Then the set $P^{-1}(0)$ is a stabilizing hypersurface in $\mathbb{C}^n$.*

Proof. Let $P = P_1^{r_1} \dots P_s^{r_s}$, where $P_1, \dots, P_s$ are pairwise different irreducible factors of P. Observe that $P' = P_1 \dots P_s$ is a test polynomial in the class of monomorphisms too. Indeed, if $\Phi : \mathbb{C}[T_1, \dots, T_n] \to \mathbb{C}[T_1, \dots, T_n]$ is a monomorphism such that $\Phi(P') = cP'$ for some $c \in \mathbb{C}^*$ then $\Phi(P_i) = c_i P_{\sigma(i)}$, where σ is a permutation of the set $\{1, \dots, s\}$ and $c_i \in \mathbb{C}^*$. Hence $\Phi^{s!}(P_i) =$

$c_i' P_i$, so $\Phi^{s!}(P) = c' P$, where $c_i', c' \in \mathbb{C}^*$. This implies that $\Phi^{s!}$ is an automorphism, and so is Φ. Therefore for this proof we may assume that $r_1 = \ldots = r_s = 1$.

Put $H = P^{-1}(0)$ and $H_i = P_i^{-1}(0)$. Suppose that H is not stabilizing. Let $f : \mathbb{C}^n \times \mathbb{C}^m \to X \times \mathbb{C}^m$ be an isomorphism such that $f(H \times \mathbb{C}^m) = H' \times \mathbb{C}^m$ and for some $x \in X$ the dimension of $\pi_{\mathbb{C}^n}(f^{-1}(\{x\} \times \mathbb{C}^m))$ is positive. Choose points $a_i \in H_i \setminus \bigcup_{j \neq i} H_j$ such that H_i is non-singular at a_i for all i. Assume that $f(a_i)$ lies on $\{b_i\} \times \mathbb{C}^m$ for some $b_i \in X$ (we identify $\mathbb{C}^n$ with $\mathbb{C}^n \times \{0\}$). By Lemma 4.5 there exists a morphism $p : \mathbb{C}^n \to \mathbb{C}^m$ such that its graph meets transversally $f^{-1}(\{b_i\} \times \mathbb{C}^m)$ at a_i and cuts $f^{-1}(\{x\} \times \mathbb{C}^m)$ in at least two points. Similarly, we can find a morphism $q : X \to \mathbb{C}^m$ with the graph meeting transversally $f(\{a_i\} \times \mathbb{C}^m)$ at $f(a_i)$ for all i. Consider the morphism $g := {f^{-1}}_q \circ f_p : \mathbb{C}^n \to \mathbb{C}^n$, where $f_p : \mathbb{C}^n \ni x \mapsto \pi_X(f(x, p(x))) \in X$ and ${f^{-1}}_q : X \ni x \mapsto \pi_{\mathbb{C}^n}(f^{-1}(x, q(x))) \in \mathbb{C}^n$. By definition, g is not injective, $g(a_i) = a_i$ and g is a local biholomorphism at a_i. In particular, g is dominant. Obviously, $g^{-1}(H) = H$. Hence for the induced monomorphism $g^* : \mathbb{C}[T_1, \ldots, T_n] \to \mathbb{C}[T_1, \ldots, T_n]$ we have $g^*(P) = c P_1^{d_1} \ldots P_s^{d_s}$ for some $c \in \mathbb{C}^*$ and $d_1, \ldots, d_s > 0$. But the tangent space to H at a_i is given by two equations $d_{a_i} P = 0$ and $d_{a_i} g^*(P) = 0$, so $d_1 = \ldots = d_s = 1$. Consequently g is an automorphism, despite g is not injective. This concludes the proof. $\square$

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