Theorem 1. *(Abhyankar-Moh)If $l = d_i$ for some $1 \leqslant i \leqslant h+1$ and $d_1 \neq d_2$ then:*

1. $\sqrt[d_i]{f}$ *is irreducible in* $\mathbb{K}((X))[Y]$,

2. *if* $2 \leqslant i \leqslant h+1$ *then for every Puiseux root* $z(t) \in \mathbb{K}\left(\left(t^{1/M}\right)\right)$, $M = k!$, *of* $\sqrt[d_i]{f}(t, Y)$ *there exists* $\varepsilon \in U_k(\mathbb{K})$ *such that*

$$\operatorname{ord}_t \left(y(\varepsilon t) - z\left(t^k\right)\right) = m_i,$$

3. *if* $2 \leqslant i \leqslant h+1$

$$\operatorname{ord}_t \left(\sqrt[d_i]{f}\left(t^k, y(t)\right)\right) = r_i.$$

1. Results

Our results can be summarized as follows.

Theorem 2. *Let* l *be integer such that* $l|k$, $l \notin \{d_1, ..., d_{h+1}\}$, $i := max\{1 \leqslant j \leqslant h+1 : l|d_j\}$. *Then:*

1. *point 1. of Theorem 1 is not true (see example below)*

2. *for every Puiseux root* $z(t) \in \mathbb{K}\left(\left(t^{1/M}\right)\right)$, $M = k!$, *of* $\sqrt[l]{f}(t, Y)$ *there exists* $\varepsilon \in U_k(\mathbb{K})$ *such that*

$$\operatorname{ord}_t\left(y(\varepsilon t) - z\left(t^k\right)\right) \geqslant m_i;$$

3.

$$\operatorname{ord}_t\left(\sqrt[l]{f}\left(t^k, y(t)\right)\right) \geqslant r_i \frac{d_i}{l}$$

If, in addition, $l > d_{i+1}$ *then the above inequalities are in fact* *equalities.*

Example 1. *Take the parametrization* $X = t^{48}$, $Y = 1/(t^{36}) + 1/(t^{6}) + 1/(t^{5})$ *and let* f *be the minimal monic polynomial for it. Then* $f = Y^{48} + \ldots$. *It can be verified that for* $l = 2$ $\sqrt[l]{f} = Y^{24} + \ldots$ *splits into three irreducible factors in* $\mathbb{C}\left(\left(X\right)\right)\left[Y\right]$ *each of them having partial Puiseux root of the form* $t^{-3/4} + t^{-1/8} + \theta\, t^{1/8} + h.o.t.$ *It's worth noticing that the divisor* $l = 2$ *here is very regular - we have* $d_4 = 1|2|d_3 = 6$ *and despite of that irreducibility does not follow.*

It is also easy to give examples in the other direction. Let $X = t^{18}, Y = t^{-12} + t^{-2} + t^{-1}$, $l = 3$ *and let* f *be the minimal monic polynomial for it. Then* $f = Y^{18} + \ldots$. *It can be verfied that* $\sqrt[l]{f}$ *is irreducible.*

Example 2. *Let* $X = t^{18}$, $Y = t^{-12} + at^{-3} + bt^{-1}$, *where* a, b *are indeterminates over* $\mathbb{C}$, $l = 2$. *Then* $\underline{l = 2 < d_{i+1} = 3}$, *so the assumption made in Theorem 2 is not fulfilled. In spite of that we have* $\operatorname{inco}_t \sqrt[l]{f}\left(t^6, 1/t^4 + Z/t\right) = -27/2 \cdot Z(-2Z^2 + 3a^2)$. *We conclude, that* $\sqrt[l]{f}$ *has two non-conjugate Puiseux roots. One of them is of the form* $z_1(t) = t^{-2/3} + \sqrt{6}/2 \cdot a \cdot t^{-1/6} + h.o.t.$ *whereas* $y(t) = t^{-12} + at^{-3} + bt^{-18}$ *so still* $\operatorname{ord}_t\left(y\left(t\right) - z_1\left(t^{18}\right)\right) = -3 = m_2$. *Also* $\operatorname{ord}_t\left(\sqrt[l]{f}\left(t^{18}, y\left(t\right)\right)\right) = r_2 \frac{d_2}{l} = -81.$*)*

Problem 1. *Can we drop the assumption* $l > d_{i+1}$*?*

Problem 2. *If* $\sqrt[l]{f}$ *is reducible in* $\mathbb{K}((X))[Y]$*, do the degrees of the factors divide* k*?*