On the duals of certain subnormal tuples

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Abstract

Part I: The tuple of multiplications by coordinate functions on the Hardy space of the open unit ball $\mathbb{B}^{2m}$ in $\mathbb{C}^m$ (resp. open unit polydisk $\mathbb{D}^m$ in $\mathbb{C}^m$) is referred to as the Szegö tuple (resp. Cauchy tuple) and is a well-known example of a subnormal operator tuple. Naturally associated with the Szegö tuple (resp. Cauchy tuple) is its dual whose coordinates act on the orthocomplement of the Hardy space of the ball (resp. polydisk) in an appropriate L^2 space. We examine the Koszul complexes associated with the duals of the Szegö and Cauchy tuples and determine their Betti numbers. We explicitly verify that, for $m \geq 2$, the m'th cohomology vector space associated with the Koszul complex of either the dual of the Szegö tuple or the dual of the Cauchy tuple is zero-dimensional. It follows in particular that, for $m \geq 2$, neither the Szegö m-tuple nor the Cauchy m-tuple is quasisimilar to its dual; this is in contrast with the case $m = 1$ where both the Szegö tuple and the Cauchy tuple reduce to the Unilateral Shift which is known to be unitarily equivalent to its dual. Part I is joint work with Pramod Patil.

Part II: We consider an important class of subnormal operator m-tuples M_p ($p = m, m+1, ...$) that is associated with a class of reproducing kernel Hilbert spaces $\mathcal{H}_p$ (with M_m being the multiplication tuple on the Hardy space of the open unit ball $\mathbb{B}^{2m}$ in $\mathbb{C}^m$ and M_{m+1} being the multiplication tuple on the Bergman space of $\mathbb{B}^{2m}$). Given any two C*-algebras $\mathcal{A}$ and $\mathcal{B}$ from the collection $\{C^*(M_p), C^*(\tilde{M}_p) : p \geq m\}$, where $C^*(M_p)$ is the unital C*-algebra generated by M_p and $C^*(\tilde{M}_p)$ the unital C*-algebra generated by the dual $\tilde{M}_p$ of M_p, we verify that $\mathcal{A}$ and $\mathcal{B}$ are either *-isomorphic or that there is no homotopy equivalence between $\mathcal{A}$ and $\mathcal{B}$. For example, while $C^*(M_m)$ and $C^*(M_{m+1})$ are well-known to be *-isomorphic, we find that $C^*(\tilde{M}_m)$ and $C^*(\tilde{M}_{m+1})$ are not even homotopy equivalent; on the other hand, $C^*(M_m)$ and $C^*(\tilde{M}_m)$ are indeed *-isomorphic.

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