Completely positive maps on Coxeter groups with applications to noncommutative von Neumann inequality, deformed Fock spaces, BMV conjecture and free probability

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Abstract

The plan of my talk is following:

1. Quasi-multiplicative operator-valued functions P on permutation (Coxeter) groups and free groups G with respect natural length functions $L(x)$ = minimal numbers of generators in the word x in G), i.e.

$$P(xy) = P(x)P(y), \text{ if } L(xy) = L(x) + L(y), \text{ for } x, y \text{ in a group } G,$$
$$P(x^{-1}) = P(x)^* \text{ and } P(e) = I.$$

2. Completely positive maps of quasi-multiplicative functions on C*-algebras of permutations, Coxeter groups and free groups.
3. Applications to noncommutative von Neumann inequality:
   For arbitrary contractions T_j on a Hilbert space and arbitrary noncommutative polynomial on n variables we have
   $\|p(T_1, ..., T_n)\| < sup\{\|p(U_1, ..., U_n)\|\}$; where U_j are finite dimensional unitary matrices.
   Case $n = 1$ is the classical von Neumann inequality.
4. For arbitrary self-adjoint contraction T,which is Yang-Baxter ,on tensor product of Hilbert spaces, HXH, we give construction of large class of deformed Fock spaces and many examples of von Neumann algebras which are factors i.e. centrum is trivial.
5. We prove for large class of self-adjoint operators A and B that the Bessis-Moussa-Villani(BMV) conjecture is true i.e.:
   The function on the real line $F(x) = tr(exp(A + ixB))$ is positive definite.
6. Free infinitely divisibility of the q-Gaussian law -the Jacobi theta one function, for which orthogonal polynomials are q-Hermite polynomials

$$xH_n(x) = H_{n+1}(x) + (1 - q^n)/(1 - q)H_{n-1}(x),$$

   for all q in interval $[0, 1]$; the case $q = 1$ is the classical NORMAL law.

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