Boundedness and compactness of Berezin-Toeplitz operators

L. A. Coburn

Abstract

For the Berezin-Toeplitz operators on the Hilbert space $H^2(\mathbf{C}^n, d\mu)$ of Gaussian square-integrable entire functions on complex n-space, $\mathbf{C}^n$, I discuss known criteria for boundedness and compactness. In this discussion the standardized Gaussian measure is given by $d\mu = (2\pi)^{-n} \exp\{-|z|^2/2\} dv(z)$. Considerable progress has been made in recent papers [CIL, BCI]. Here, B = Wolfram Bauer, I = Josh Isralowitz, L = Bo Li. For g in $L^2(\mathbf{C}^n, d\mu)$ and $s > 0$, we consider the "heat transform"

$$(*) \qquad \widetilde{g}^{(s)}(a) = \int_{\mathbf{C}^n} g(w) exp\{-|w-a|^2/4s\} dv(w)(4\pi s)^{-n}$$

In [CIL], we showed that, for g in BMO^1, the Berezin-Toeplitz operator T_g is bounded (compact) if and only if the *Berezin transform* $\widetilde{g} \equiv \widetilde{g}^{(1/2)}$ is bounded (vanishes at infinity). In [BCI], we used the results of [CIL] and a refined version of results with Charles Berger in [BC_2] to show, again for g in BMO^1, that the following three statements are equivalent: (a)T_g is compact, (b) $\widetilde{g}^{(t_0)}$ vanishes at infinity (is in $C_0(\mathbf{C}^n)$) for *some* $t_0 > 0$ and (c) $\widetilde{g}^{(t)}$ is in $C_0(\mathbf{C}^n)$ for *all* $t > 0$.

State University of New York, Buffalo, NY, USA
E-mail address: `lcoburn@buffalo.edu`