Schrödinger operators with δ'-like potentials

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Abstract

We address the problem on the right definition of the Schrödinger operator with potential δ', where δ is the Dirac delta-function. Namely, for a real-valued function V from the Faddeev–Marchenko class $L_1\big(\mathbb{R}, (1+|x|)dx\big)$, we show the norm resolvent convergence, as $\epsilon \to 0$, of a family S_ϵ of one-dimensional Schrödinger operators on the line of the form

$$S_\epsilon := -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{\epsilon^2} V\Big(\frac{x}{\epsilon}\Big).$$

If the potential V satisfies the conditions

$$\int_{\mathbb{R}} V(\xi)\,\mathrm{d}\xi = 0, \qquad \int_{\mathbb{R}} \xi V(\xi)\,\mathrm{d}\xi = -1,$$

then the functions $\epsilon^{-2}V(x/\epsilon)$ converge in the sense of distributions as $\epsilon \to 0$ to $\delta'(x)$, and the limit S_0 of S_ϵ might be considered as a 'physically motivated' interpretation of the one-dimensional Schrödinger operator with potential δ'.

In 1986, P. Seba claimed that the limit coincides with the direct sum of free Schrödinger operators on the semi-axes with the Dirichlet boundary condition at the origin, which implies that in dimension one there is no non-trivial Hamiltonians with potential δ'. Our results demonstrate that, although the above statement is true for many V, for the so-called resonant V the limit operator is defined by the non-trivial interface condition at the origin determined by some spectral characteristics of V. In this resonant case, we show that there is a partial transmission of the wave package for the limiting Hamiltonian.

Recently similar results on norm resolvent convergence were established by S. Albeverio, C. Cacciapuoti, P. Exner and D. Finco under the assumptions that V decays exponentially and its mean is nonzero, which, however, excludes the important case of δ'-like potentials.

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