Selfadjoint operators in S-Spaces

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Abstract

Let $\mathfrak{S}$ be a complex vector space and let $[\cdot,-]$ be a sesquilinear form on $\mathfrak{S}$. The pair $(\mathfrak{S},[\cdot,-])$ is said to be an *S-Space* if there is a Hilbert space scalar product $(\cdot,-)$ on $\mathfrak{S}$ and a linear operator U in $\mathfrak{S}$ which is unitary with respect to $(\cdot,-)$ such that

$$[x,y]=(Ux,y)\quad \text{for all}\ \ x,y\in\mathfrak{S}.$$

The pair $(U,(\cdot,\cdot))$ will be called a *Hilbert space realization* of the S-Space $(\mathfrak{S},[\cdot,-])$. Note that this can be seen as a generalization of the concept of Krein space (where $U=U^*$).

We show that two Hilbert space realizations $(U_1,(\cdot,-)_1)$ and $(U_2,(\cdot,-)_2)$ are equivalent in the sense that $(\cdot,-)_1$ and $(\cdot,-)_2$ induce the same topology on $\mathfrak{S}$ and that U_1 and U_2 are similar operators.

After examining such fundamental properties of S-Spaces we consider closed and densely defined linear operators in $\mathfrak{S}$. Since $[\cdot,-]$ is in general non-Hermitian there are two adjoints $A^\natural$ and ${}^\natural A$ for such an operator A. The operator A is called *selfadjoint* in the S-Space $(\mathfrak{S},[\cdot,-])$ if $A=A^\natural$. It is shown that this is equivalent to $A={}^\natural A$.

We show that for a selfadjoint operator A in the S-Space $(\mathfrak{S},[\cdot,-])$ the spectral subspaces of U^2 are A-invariant. Hence, since in the Krein space case (i.e. if $U=U^*$) we have $U^2=I$, this does not give any information on A whereas in the general S-Space situation (i.e. $U\neq U^*$) we automatically have non-trivial invariant subspaces of A.

However, we prove that any selfadjoint operator in the S-Space $(\mathfrak{S},[\cdot,-])$ is also selfadjoint with respect to some Krein space inner product $\langle\cdot,-\rangle$ on $\mathfrak{S}$.

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