Operator realizations of certain topological quasi *-algebras

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Abstract

Partial *-algebras of unbounded operators defined on a domain $\mathcal{D}$ in Hilbert space $\mathcal{H}$ (shortly, partial O*-algebras) are the natural arena where representing locally convex quasi *-algebras. These latter typically arise when completing a given locally convex *-algebra $\mathfrak{A}_0[\tau]$ with respect to a topology τ, when the multiplication is not jointly continuous. After a survey of the basic aspects of this matter, we will focus our attention to the case where $\mathfrak{A}_0$ is a C*-algebra in itself and so the interplay between the two topologies provides more information on the structure of this quasi *-algebra.

More precisely, consider a C*-algebra $\mathfrak{A}_0$ and suppose that $\mathfrak{A}_0$ is also endowed with a locally convex topology τ (coarser than the norm topology), so that $\mathfrak{A}_0$ is a locally convex *-algebra with continuous involution and *separately continuous* multiplication. The completion $\mathfrak{A}$ of $\mathfrak{A}_0$ under this topology is, in general, not an algebra but only a quasi *-algebra, since the multiplication of two elements of $\mathfrak{A}$ need not be defined. Under certain *regularity conditions*, this quasi *-algebra attains a richer structure, called *locally convex quasi C*-algebra*, where some of the well known properties of C*-algebras extends in rather natural way (for instance, the functional calculus for positive elements). After presenting some basic examples, we will discuss conditions for a locally convex quasi C*-algebra $(\mathfrak{A}[\tau], \mathfrak{A}_0)$ to possess a sufficiently large family of *-representations. In particular, two results (the first one refers to the commutative case and the second to the noncommutative one) give a deeper insight on the structure of locally convex quasi C*-algebras, provided that the family of invariant positive sesquilinear (ips-) forms on $\mathfrak{A}$ is itself so large to *separate points* of $\mathfrak{A}$. Some of these results can also be extended to the case where $\mathfrak{A}_0$ is only a C*-normed algebra (i.e. not necessarily complete).

Moving in the opposite direction, we will consider a locally convex quasi *-algebra $(\mathfrak{A}[\tau], \mathfrak{A}_0)$ having sufficiently many representations and look for conditions under which the set of its *bounded elements* (roughly speaking, elements whose image under every *-representation is a bounded operator) is a C*-normed algebra, dense in $\mathfrak{A}[\tau]$.

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