Equisingularity in families of curves and surfaces

Otoniel Nogueira da Silva *

Abstract

Consider a surface $X \in \mathbb{C}^n$ and a generic projection $p : X \to T$, where T is a neighborhood of 0 in $\mathbb{C}$. We can see X as a family of curves with fibers $X_t = p^{-1}(t)$ (in other words, a deformation of the special fiber $X_0 = p^{-1}(0)$). When $t \neq 0$ is generic, we say that $X_t = p^{-1}(t)$ is a generic fiber of p (see Figure 1). We have the following natural question:

Question: How can we compare (in some sense) the singularities of the generic and special fiber?

In general terms, that's the idea of equisingularity theory, i.e., how to compare the singularities that appear in the family X_t. In this talk, we will speak about invariants that control some types of equisingularity of X. When X is not Cohen-Macaulay the special fiber X_0 has a "fat point" at the origin. So, since X_0 is no longer reduced, it seems that the classical invariants like the Milnor number cannot be applied directly. Hence, we need new invariants to control the equisingularity of X.

In this context, we will present a substitute for the Milnor number which is an invariant developed by Greuel and Brucker. Finally, we will relate this invariant to the equisingularity of X.

This is a joint work with J. Snoussi.

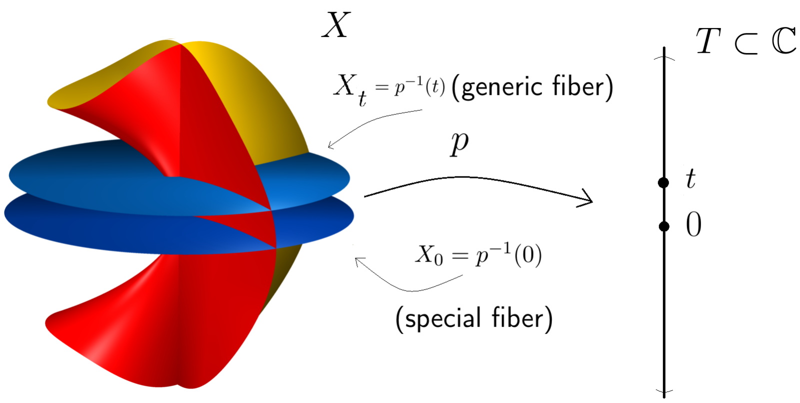

Figure 1: The family of curves $p : X \to T$.

*O.N. Silva: Departamento de Matemática, Universidade Federal de São Carlos, Caixa Postal 676, 13560-905 São Carlos, SP, Brazil, e-mail: otoniel@dm.ufscar.br