Deformations of structures to geometric black hole of a compact manifold

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Let M^n be a compact, closed, smooth manifold. Then there exists a Riemannian metric g on M^n and a smooth triangulation of M^n. Both the structures make possible to construct a family of piecewise smooth curves on M^n and to prove the following

Theorem 1. *The manifold* M^n *has a decomposition* $M^n = C^n \cup K^{n-1}$, $C^n \cap K^{n-1} = \emptyset$, *where* C^n *is a ndimensional cell and* K^{n-1} *is a connected union of some* $(n-1)$ *simplexes of the triangulation.*

Using the construction above we have obtained

Theorem 2. (Poincare). *Let* M^3 *be a compact, closed, smooth, simply connected manifold of dimension* 3*. Then* M^3 *is diffeomorphic to* S^3*, where* S^3 *is the sphere of dimension* 3*.*

For any point $z \in K^{n-1}$ we can consider the closed geodesic ball $\overline{B}(z, \varepsilon)$ of a small radius $\varepsilon > 0$. Let $Tb(K^{n-1}, \varepsilon) = \bigcup_{z \in K^{n-1}} \overline{B}(z, \varepsilon) = BH(\varepsilon)$.

Definition. We call the set $BH(\varepsilon)$ a geometric black hole of radius $\varepsilon > 0$ of the manifold M^n if $M^n \backslash BH(\varepsilon)$ is a cell (it is true for some small ε).

Let T be some structure (tensor field, Gstructure *etc*) on the manifold M^n. Deformations of structures was considered in [1].

Thus, there exists a deformation $\overline{T}$ of T on M^n with the following properties.

a) T is continuous and sectionally smooth.

b) If a point $x \in M^n \backslash BH(\varepsilon)$ then the construction of $\overline{T}$ at the point x is trivial.

1. A.A. Ermolitski. Deformations of structures, embedding of a Riemannian manifold in a Kaehlerian one and geometric antigravitation. Banach Center Publications, V. 76, p. 505-514, Warszawa 2007.