**Euler class and Gysin sequence of the oriented sphere bundle on differential spaces**

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Let $(M, \pi, B, F)$ be an oriented, Riemannian $(r+1)$-vector bundle, where $(M, \mathcal{F}(M))$ is the differential space (in the sense of Sikorski), with a differential structure $\mathcal{F}(M)$, $(B, \mathcal{F}(B))$ is a base differential space, $\pi : M \to B$ is the projection. With such vector bundle we can associate an oriented $r$-sphere bundle $(M_S, \pi_S, B, S)$, where $(M_S, \mathcal{F}(M_S))$ is differential subspace of $(M, \mathcal{F}(M))$ with the same differential structure, $\pi_S : M_S \to B$ is the restriction of $\pi$, $S$ (resp. $S_x$) denotes the unit sphere of the vector space $F$ (resp. $F_x$) and

$$M_S = \bigcup_{x \in B} S_x.$$

We define integration over the fibre $F$ as a linear map $\fint_F : A_F(M) \to A(B)$,homogenous of degree $-r-1$, where $A_F(M)$ denotes the set of forms with fibre-compact support. We show following properties of the integration over the fibre:

1. $\fint_F$ is surjection;
2. $d \circ \fint_F = \fint_F \circ d$, where $d$ is differential operator;
3. $\fint_F \pi^*\Phi \wedge \Psi = \Phi \wedge \fint_F \Psi$, where $\Phi \in A(B)$, $\Psi \in A_F(M)$, $\pi^* : A(B) \to A(M)$.

The aim of this work is to construct the Gysin sequence and the Euler class of the oriented sphere bundle on differential space. We need to this construction a homomorphism between cohomology algebras $\beta : H(B) \to H(\ker \fint_S)$, giving by the formula: $\beta([\Phi]) = [\pi^*\Phi]$ for $[\Phi] \in H(B)$. We show in this paper that $\beta$ is an isomorphism and next we construct the Gysin sequence for the sphere bundle and the Euler class of the oriented sphere bundle on a differential space.