On the structure of factorizable groups of homeomorphisms

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Let $G(X)$ be a group of homeomorphisms on a paracompact second countable space X. $G(X)$ is called *factorizable* if for any open cover $\mathcal{U}$ of X the set $\bigcup_{U \in \mathcal{U}} G_U(X)$ generates $G(X)$, where $G_U(X) = \{f \in G(X) : \mathrm{supp}(f) \subset U\}$.
A well-known Epstein's theorem states that under some natural conditions (including the transitivity and the factorizablility) the commutator subgroup $[G(X), G(X)]$ of $G(X)$ is simple. Next Ling ameliorated the Epstein's theorem by relaxing its assumptions (but the transitivity is still assumed).

In this talk we describe the algebraic properties of the commutator subgroup of a homeomorphism group $G(X)$, which need not satisfy transitivity assumption.

Definition: Let G acts effectively on X, i.e. G is a subset of the symmetric group of X. A group G is said to be *non-fixing* if $\mathcal{O}(x) \neq \{x\}$ for every $x \in X$, where $\mathcal{O}(x) = \{g(x) | g \in G\}$ is an orbit of x.
A non-fixing group G is said to be *quasi-simple* iff there is no nontrivial, non-fixing normal subgroup of G.

Let us introduce the following condition of *strong factorizability* for a group $G = G(X)$ of homeomorphisms of X.
(**SF**) For any $g \in G$ and for any $\mathcal{V}$ a covering of X there exist an integer n, elements $g_1, \ldots, g_n \in G$ and $V_1, ..., V_n \in \mathcal{V}$ such that $g = g_n \ldots g_1$ and $\mathrm{supp}(g_i) \subseteq V_i$. Moreover for any $x \in X$ there is U an open neighbourhood of x such that if $\mathrm{supp}(g) \subseteq U$ then we may have $\mathrm{supp}(g_i) \subseteq U$ for $1 \leq i \leq n$.

Our main result could be formulated as follows.

Theorem: Let X be a paracompact space and let $G = G(X)$ satisfy the strong factorizability condition (SF). If H is a non-fixing subgroup of G such that $[G, G] \subseteq \mathcal{N}_G(H)$ then $[G, G] \subseteq H$.

Several examples illustrating the theorem are presented. For instance, the compactly supported identity component of the group of all leaf-preserving diffeomorphisms of a regular foliation is quasi-simple. The same is true for the diffeomorphism group of a manifold with boundary.

References

[1] D.B.A. Epstein: *The simplicity of certain groups of homeomorphisms*, Compositio Mathematica 22, Fasc.2 (1970), 165-173

[2] W.Ling, *Factorizable groups of homeomorphisms*, Compositio Mathematica, 51 no. 1 (1984), 41-50