A reconstruction theorem for smooth foliated manifolds

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Let $1 \leq k \leq \infty$ and X be a second countable C^k foliated manifold. Denote by $H_1^k(X)$ the group of all C^k auto-homeomorphisms of X which take every leaf of X to a leaf of X. Denote by $H_0^k(X)$ the group of all members of $H_1^k(X)$ which take every leaf of X to itself.

Theorem Let $1 \leq k \leq \infty$ and X_1, X_2 be second countable C^k foliated manifolds. For $i = 1, 2$ let G_i be subgroups such that $H_0^k(X_i) \subseteq G_i \subseteq H_1^k(X_i)$. Suppose that φ is an isomorphism between G_1 and G_2. <u>Then</u> there is a homeomorphism τ between X_1 and X_2 such that: (i) $\varphi(g) = \tau \circ g \circ \tau^{-1}$ for every $g \in G_1$, and (ii) τ takes every leaf of X_1 to a leaf of X_2.