

Global smooth fibrations of \mathbb{R}^3 by oriented lines

Marcos Salvai

A smooth fibration of \mathbb{R}^3 by oriented lines is given by a smooth unit vector field V on \mathbb{R}^3 all of whose integral curves are straight lines. Such a fibration is said to be nondegenerate if dV vanishes only in the direction of V . Let \mathcal{L} be the space of oriented lines of \mathbb{R}^3 endowed with its canonical pseudo-Riemannian neutral metric. We characterize the nondegenerate smooth fibrations of \mathbb{R}^3 by oriented lines as the definite closed (in the relative topology) connected surfaces in \mathcal{L} . In particular, local conditions on \mathcal{L} imply the existence of a global fibration. Besides, for any such fibration, the base space is diffeomorphic to the open disc and the directions of the fibers form an open convex set of the two-sphere. We characterize as well, in a similar way, the smooth (possibly degenerate) global fibrations.

The spirit of the article is similar to that of the characterization given by H. Gluck and F. Warner (Duke Math. J., 1983) of the oriented great circle fibrations of S^3 : They determine which subsets of the manifold of oriented circles $C \cong S^2 \times S^2$ are base spaces of such fibrations. In the Euclidean situation, by contrast, apart from the noncompactness of the ambient space, one has the difficulties arising from the fact that \mathcal{L} is not Riemannian and from the possibility of degeneracy (existence of infinitesimally parallel fibers, not present in the spherical case).