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Let ABC be a triangle and $A_1 \in BC$, $B_1 \in AC$, $C_1 \in AB$ be such that $AA_1 \cap BB_1 \cap CC_1 \neq \emptyset$. Let us denote $AA_1 \cap BB_1 \cap CC_1 = M$.

Problem 1 *Let ABC be a equilateral triangle. Prove that:*

$$A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq A_1B \cdot B_1C \cdot C_1A.$$

Problem 2 *Let us consider a square of area 1 with n points inside. Prove that one can number this points by $P_1, P_2, \ldots, P_n$ such that:*

$$|P_1P_2|^2 + |P_2P_3|^2 + \cdots + |P_nP_1|^2 \leq 4.$$

Problem 3 *Let $ABCDEF$ be a hexagon such that*

$$AB = BC, \quad CD = DE, \quad EF = FA.$$

Prove that

$$\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geq \frac{3}{2}.$$

Problem 4 *Let $ABCDEF$ be a hexagon such that*

$$\angle ABC = \angle BCD = \angle CDE = \angle DEF = \angle EFA = \angle FAB.$$

Prove that

$$AB + BC = DE + EF.$$

Problem 5 *Let A, B, C, D, E lie on a semicircle with radius 1.*

Prove that

$$AB^2 + BC^2 + CD^2 + DE^2 + AB \cdot BC \cdot CD + BC \cdot CD \cdot DE < 4.$$

Problem 6 *Let $A_1, A_2, \ldots A_n$ be points on a circle (with radius 1 and center O) such that*

$$\overrightarrow{OA_1} + \overrightarrow{OA_2} + \cdots + \overrightarrow{OA_n} = 0.$$

Prove that for any point B

$$A_1B + A_2B + \ldots A_nB \geq n.$$

Problem 7 *Every point on the plane has been colored by blue or yellow.*

Prove that there is a equilateral triangle with all vertices colored by the same colour.

Problem 8 *Every point in the space has been colored by blue or yellow.*

Prove that there is a square with sides equal 1, such that has 0,1 or 4 yellow vertices.

Problem 9 *Let $\Omega_1 \cup \Omega_2 \cup \Omega_3 = \mathbb{C}$ such that $\Omega_i \cap \Omega_j = \emptyset$, for $i \neq j$.*

Prove that at least one set Ω_i has the following property:

for any $d > 0$ there are $x, y \in \Omega_i$ such that $|x - y| = d$.

Problem 10 *It is possible to find two tetrahedrons Γ_1, Γ_2 such that $\Gamma_1 \subset \Gamma_2$, but the sum of lengths of edges of Γ_1 is greater than the sum of lengths of edges of Γ_2.*

Problem 11 *In a disc (with radius* 1*) there are* 64 *points. Prove that* 10 *of them lies in some disc with radius* $\frac{1}{2}$*.*

Problem 12 *In each square of the chessboard* 2021×2021 *there is a number with modulus less than* 1*. Moreover, the sum of numbers from any square* 2×2 *is equal* 0*. Show that the sum of all numbers is less than* 2021*.*

Problem 13 *Can we cover a rectangle using* 100 *balls with radius* 1*, if we know that we can do that by using* 25 *balls with radius* 2*.*

Problem 14 *The three canonical projections of some convex set* $\Gamma \subset \mathbb{R}^3$ *are circles with radius* 1*. Is* Γ *a ball?*