Geometry of Banach Spaces and Related Topics

The Anniversary of Professor Grzegorz Lewicki

8-10 June 2017, Kraków, Poland

Program

Thursday, June 8th, 2017:

09:00-10:00 Registration

10:00-10:10 Opening ceremony (room 0094)

10:10-11:10 Lesław Skrzypek *On the Maximal (and almost Maximal) Relative Projection Constants*

11:20-12:20 Anna Kamińska *Diameter two properties in Banach spaces*

12:20-13:00 Coffee break

13:00-13:30 Michael Prophet *Some results on the existence and minimality of shape-preserving extentions*

13:35-14:05 Maciej Ciesielski *Hardy-Littlewood-Polya relation in the best dominated approximation in symmetric spaces*

14:10-14:40 Malgorzata Stawiska-Friedland *Completeness of certain spaces of probability measures on $\mathbb{R}^N$ in a Fourier-based metrics*

15:00-16:00 Lunch break

16:00-18:00 Łojasiewicz Lecture (room 0004) /Table tennis practice (room 1083)

18:00-20:00 Table tennis competition (room 1083)

Friday, June 9th, 2017:

10:00-11:00 Henryk Hudzik *Order asymptotically isometric copies of l^∞, c_0 and l^1 in Kothe spaces*

11:10-12:10 Michał Wojciechowski *On the Pełczyński conjecture on Auerbach bases*

12:10-12:40 Coffee break

12:40-13:10 Anna Pelczar-Barwacz *An unconditionally saturated Banach space with the scalar-plus-compact property*

13:15-13:45 Marian Nowak *Applications of the theory of Orlicz spaces to vector measures*

13:50-14:20 Alberto Castejón and Alberto Martín Mendez *On some algebraic aspects related to the projection constants*

14:20-15:20 Lunch break

15:20-16:20 Mieczysław Mastyło *Mahler's measure of polynomials and polynomial inequalities via Rademacher processes*

Saturday, June 8th, 2017:

10:00-11:00 Asuman Aksoy *From Bernstein Pairs to the Speed of Convergence*

11:10-11:40 Tadeusz Chawziuk *Composition operator between Orlicz spaces*

11:45-12:15 Paweł Kolwicz *Local structure in Banach function spaces useful in applications to approximation problems*

12:15-12:50 Coffee break

12:50-13:20 Karol Leśnik *Toeplitz and Hankel operators acting between distinct Hardy spaces*

13:25-13:55 Piotr Niemiec *Projections commuting with operators*

14:00-14:30 Radosław Kaczmarek *Normed Orlicz function spaces which can be quasi-renormed with easily calculable quasi-norms*

15:00-18:00 Conference banquet (1016)

18:00-19:30 Open discussion session

FROM BERNSTEIN PAIRS TO THE SPEED OF CONVERGENCE

ASUMAN GÜVEN AKSOY, GRZEGORZ LEWICKI

Abstract

One of the notable theorems used in the constructive theory of functions is Bernstein Lethargy Theorem (BLT). Many years ago, G. Lewicki and I identified some classical Banach spaces that form a Bernstein pair [1]. Later, by using a theorem of Borodin from [4], we were able to extend Bernstein's Lethargy Theorem (BLT) [3] to Fréchet spaces [2]. More precisely, let X be an infinite-dimensional Fréchet space and let $\mathcal{V} = \{V_n\}$ be a nested sequence of subspaces of X such that $\overline{V_n} \subseteq V_{n+1}$ for any $n \in \mathbb{N}$. Let e_n be a decreasing sequence of positive numbers tending to 0. Under an additional natural condition on $\sup\{\operatorname{dist}(x, V_n)\}$, we prove there exists $x \in X$ and $n_o \in \mathbb{N}$ such that

$$\frac{e_n}{3} \leq \operatorname{dist}(x, V_n) \leq 3e_n$$

for any $n \geq n_o$. By using the above theorem, we prove both Shapiro's [6] and Tyuremskikh's [7] theorems for Fréchet spaces. We also give a theorem improving Konyagin's [5] result for Banach spaces.

Keywords: Best Approximation, Bernstein's Lethargy Theorem, Fréchet Spaces.
AMS Classification: 41A25, 41A50, 41A65.

Bibliography

[1] A. G. Aksoy and G. Lewicki, *Diagonal operators, s-numbers, and Bernstein pairs*, Note di Matematica, 17, 209-216, (1999).

[2] A. G. Aksoy and G. Lewicki, *Bernstein's Lethargy Theorem in Fréchet Spaces*, J. of Approx. Theory, 209, pp 58-77, (2016).

[3] S. N. Bernstein, *On the inverse problem in the theory of best approximation of continuous functions*, Collected works (in Russian), vol II, Izd. Akad. Nauk, USSR, 1954, pp 292-294.

[4] P. A. Borodin, *On the existence of an element with given deviations from an expanding system of subspaces*, Mathematical Notes, Vol. 80, No. 5, 621-630, (2006). (Translated from Mathematicheskie Zametki).

[5] S. V. Konyagin, *Deviation of elements of a Banach space from a system of subspaces*, Proceedings of the Steklov Institute of Mathematics, 2014, Vol. 284, 204-207, (2014). (Translated from Matematicheskogo Instituta imeni V.A. Steklova).

[6] H. S. Shapiro, *Some negative theorems of approximation theory*, Michigan Math. J. 11, 211-217 (1964).

[7] I. S. Tyuremskikh, *On one problem of S. N. Bernstein*, Scientific Proceedings of Kaliningrad State Pedagog. Inst. 52, 123-129 (1967).

Asuman Güven Aksoy,
Claremont McKenna College,
Department of Mathematical Sciences,
Claremont, CA 91711, USA .
aaksoy@cmc.edu

Grzegorz Lewicki,
Jagiellonian University,
Department of Mathematics,
30-348 Krakw, Poland.
Grzegorz.Lewicki@im.uj.edu.pl

ON SOME ALGEBRAIC ASPECTS RELATED TO THE PROJECTION CONSTANTS

ALBERTO CASTEJÓN AND ALBERTO MARTÍN MENDEZ

Abstract

Let $\mathcal{U}_n$ be the set of all subspaces in $\mathbf{R}^n$ and let $\mathcal{B}_n$ be the set of all $n \times n$ real symmetric and idempotent matrices. We construct a bijective map $\Gamma\colon \mathcal{U}_n \to \mathcal{B}_n$ whose restriction to the set $\mathcal{U}_{n,k}$, $k \leq n$, of k-dimensional subspaces in $\mathbf{R}^n$ gives us a bijection onto the set $\mathcal{B}_{n,k}$ of $n \times n$ symmetric matrices with rank k (or trace k). We also give a characterization of the coefficients λ_k^n associated to the projection constant, using the spectral radius of matrices in $\mathcal{B}_{n,k}$. Finally we describe projections onto subspaces of $\mathbf{R}^n$ in terms of lateral inverses, generalizing the fact that an orthogonal projection in $\mathbf{R}^n$ is given by the product of a matrix and its pseudoinverse.

Alberto Castejón,
University of Vigo,
Department of Applied Mathematics I,
36310 Vigo, Spain
acaste@uvigo.es

Alberto Martín Méndez,
University of Vigo,
Department of Applied Mathematics II,
36310 Vigo, Spain
amartin@dma.uvigo.es

COMPOSITION OPERATOR BETWEEN ORLICZ SPACES

TADEUSZ CHAWZIUK

Abstract

Continuing on the work on the composition operator C_T acting in Orlicz space $L^{\Phi}(\Omega)$ simple and easily verifiable necessary conditions and sufficient conditions for the boundedness of the composition operator C_T acting between distinct Orlicz spaces $L^{\Phi_1}(\Omega)$ and $L^{\Phi_2}(\Omega)$ over arbitrary complete, σ-finite measure space (Ω, Σ, μ) are presented. Also, the conditions on the generating Young functions and/or the function $h = d(\mu \circ T^{-1})/d\mu$ under which the operator C_T is of closed range or finite rank are considered. Finally, necessary and sufficient conditions for the boundedness of the operator C_T is given in terms of techniques developed in the framework of the theory of Musielak-Orlicz spaces.

Bibliography

[1] T. Chawziuk, Y.Estaremi, H.Hudzik. et al. Rev Mat Complut (2017) 30: 335. doi:10.1007/s13163-016-0214-1

Tadeusz Chawziuk
Adam Mickiewicz University in Poznań,
Faculty of Mathematics and Computer Science,
61-001 Poznań, Poland
tchawz@gmail.com

HARDY-LITTLEWOOD-PÓLYA RELATION IN THE BEST DOMINATED APPROXIMATION IN SYMMETRIC SPACES

MACIEJ CIESIELSKI

Abstract

Let L^0 be a set of all (equivalence classes of) extended real valued m-measurable functions on $I = [0, \alpha)$, where $0 < \alpha \leq \infty$. For any $x \in L^0$ we denote $x^*(t) = \inf\{\lambda > 0 : m(|x| > \lambda) \leq t\}$, $x^{**}(t) = \frac{1}{t}\int_0^t x^*(s)\, ds$ for $t > 0$. The Hardy-Littlewood-Pólya relation $\prec$ is given for any x, y in $L^1 + L^\infty$ by

$$x \prec y \Leftrightarrow x^{**}(t) \leq y^{**}(t) \text{ for all } t > 0.$$

Let $(E, \|\cdot\|_E)$ be a symmetric space and let $Y \subset X$ be a nonempty subset. For $x \in X$ denote

$$P_Y(x) := \{y \in Y : \|x - y\| = dist(x, Y)\}.$$

Any element $y \in P_Y(x)$ is called a best approximant in Y to x. A nonempty set $Y \subset X$ is called *proximinal* or *set of existence* if $P_Y(x) \neq \emptyset$ for any $x \in X$. A nonempty set Y is said to be a *Chebyshev set* if it is proximinal and $P_Y(x)$ is a singleton for any $x \in E$.

A symmetric space E is said to be *strictly K-monotone* (shortly $E \in (SKM)$) if for any $x, y \in E$ such that $x^* \neq y^*$, $x \prec y$ we have $\|x\|_E \leq \|y\|_E$. A point $x \in E$ is called a point of *K-order continuity* of E if for any $(x_n) \subset E$ such that $x_n \prec x$ and ${x_n}^* \to 0$ a.e. we have $\|x_n\|_E \to 0$. A symmetric space E is called *K-order continuous* (shortly $E \in (KOC)$) if every element x of E is a point of K-order continuity.

We present results devoted to application of strict K-monotonicity and K-order continuity in symmetric spaces. We characterize a relationship between strict K- monotonicity, K-order continuity and the best dominated approximation problems with respect to the Hardy-Littlewood-Pólya relation $\prec$. First, using a local approach to strict K-monotonicity we show a necessary condition for uniqueness of the best dominated approximation under the relation $\prec$ in a symmetric space E. Next, we discuss a correlation between a point of K-order continuity and an existence of a best dominated approximant with respect to $\prec$. Finally, we present a full criteria, written in terms of K-order continuity, under which a closed and K-bounded above subset of a symmetric space E is proximinal. The above results come from the paper [1].

Bibliography

[1] M. Ciesielski, *Hardy-Littlewood-Pólya relation in the best dominated approximationin symmetric spaces* J. Approx. Theory 213 (2017), 78-91.

Maciej Ciesielski,
Poznań University of Technology,
Institute of Mathematics,
60-965 Poznań, Poland.
maciej.ciesielski@put.poznan.pl

ORDER ASYMPTOTICALLY ISOMETRIC COPIES OF l^∞, c_0 AND l^1 IN KÖTHE SPACES

HENRYK HUDZIK

Abstract

We start with the result that a Köthe space E contains an order asymptotically isometric copy of l^∞ if and only if it contains an order isometric copy of l^∞. Under suitable assumptions on a Köthe space E and its subspace E_a of order continuous elements in E, it will be shown that E_a contains an asymptotically isometric copy of c_0 whenever E contains an order isometric copy of l^∞, and vice versa, E contains an order isometric copy of l^∞ whenever E_a contains an order asymptotically isometric copy of c_0.

Moreover, under suitable assumptions on a Köthe space E and its dual space E^*, we will show that E contains an order asymptotically isometric copy of l^1 (resp. E^* contains an order isometric copy of l^∞) whenever E^* contains an order isometric copy of l^∞ (resp. E contains an order asymptotically isometric copy of l^1).

Finally, interpretations of these results in Orlicz spaces, Musielak-Orlicz spaces, Calderón-Lozanovskiĭ spaces, and Orlicz-Lorentz spaces will be given.

Henryk Hudzik,
Adam Mickiewicz University in Poznań,
Faculty of Mathematics and Computer Science,
61-001 Poznań, Poland
hudzik@amu.edu.pl

NORMED ORLICZ FUNCTION SPACES WHICH CAN BE QUASI-RENORMED WITH EASILY CALCULABLE QUASI-NORMS

RADOSŁAW KACZMAREK

Abstract

We will be interested in a widest possible class of Orlicz functions Φ such that the easily calculable quasi-norm $[f]_{\Phi,p} := \|f\|_E \left\{ I_\Phi \left(\frac{f}{\|f\|_E} \right) \right\}^{1/p}$ if $f \neq 0$ and $[f]_{\Phi,p} = 0$ if $f = 0$, on the Orlicz space $L^\Phi(\Omega, \Sigma, \mu)$ generated by Φ, is equivalent to the Luxemburg norm $\|\cdot\|_\Phi$. To do this, we will use a suitable Δ_2-condition, lower and upper Simonenko indices $p_S^a(\Phi)$ and $q_S^a(\Phi)$ for the generating function Φ, numbers $p \in [1, p_S^a(\Phi)]$ satisfying $q_S^a(\Phi) - p \leq 1$, and an embedding of $L^\Phi(\Omega, \Sigma, \mu)$ into a suitable Köthe function space $E = E(\Omega, \Sigma, \mu)$. We will take as E the Lebesgue spaces $L^r(\Omega, \Sigma, \mu)$ with $r \in [1, p_S^l(\Phi)]$, when the measure μ is non-atomic and finite, and the weighted Lebesgue spaces $L^r_\omega(\Omega, \Sigma, \mu)$, with $r \in [1, p_S^a(\Phi)]$ and a suitable weight function ω, when the measure μ is non-atomic infinite but σ- finite. We will also use condition ∇_3 if $p_S^a(\Phi) = 1$ and condition ∇^2 if $p_S^a(\Phi) > 1$, proving their necessity in most of the considered cases.

We are inspired by two examples of easily calculable quasi-norms in Orlicz spaces equivalent to the Luxemburg norm, presented by Iwaniec and Verde in 1999, and by Krbec and Schmeisser in 2012.

Our results seem important for applications of the Orlicz function spaces.

Bibliography

[1] P. Foralewski, H. Hudzik, R. Kaczmarek and M. Krbec, *Normed Orlicz function spaces which can be quasi-renormed with easily calculable quasi-norms,* to appear in Banach J. Math. Anal. in 2017.

Radosław Kaczmarek,
Adam Mickiewicz University in Poznań,
Faculty of Mathematics and Computer Science,
61-001 Poznań, Poland
radekk@amu.edu.pl

DIAMETER TWO PROPERTIES IN BANACH SPACES

ANNA KAMIŃSKA

Abstract

We will discuss different forms of diameter two property in Banach spaces. We then illustrate them in particular function spaces.

Anna Kamińska,
The University of Memphis,
Department of Mathematical Sciences,
Memphis, TN 38152, USA.
kaminska@memphis.edu

LOCAL STRUCTURE IN BANACH FUNCTION SPACES USEFUL IN APPLICATIONS TO APPROXIMATION PROBLEMS

PAWEŁ KOLWICZ

Abstract

Suppose E is a Banach lattice and $K \subset E$ is a sublattice that is K is closed with respect to finite suprema and infima (K does not need to be a linear subspace). The order interval $[u, v]$ is a typical example of a sublattice. The notation $f \leqslant K$for $f \in E$ means that $f \leqslant g$ for any $g \in K$ Given the system $f \leqslant K$ ($f \geqslant K$) set

$$P_K(f) = \left\{ u \in K : \|u - f\| = \inf_{w \in K} \|w - f\| \right\}.$$

It is known that:

(i) For all closed sublattices K and all $f \leqslant K\,(f \geqslant K)$ the set $P_K(f)$ is nonempty if and only if E is order continuous.

(ii) For all sublattices K and all $f \leqslant K\,(f \geqslant K)$ the set$P_K(f)$ is at most a singleton if and only if E is strictly monotone.

The studies of global properties are not always sufficient. When the Banach space (Banach lattice) has not the global property then it is natural to ask about the local structure. This leads, in the context of above properties, to the notion of a point of order continuity, a point of lower and upper monotonicity. We will discuss the role of these points in the local best dominated problems. We also shows that H_g-points (points of Kadec-Klee property with respect to global convergence in measure) are useful in that area (similarly as points of order continuity). Next, we consider symmetric spaces. Then a natural question is whether a point $x \in E$ has some local property if and only if its nonincreasing rearrangement x^* has the same property. We discuss this problem for a point of order continuity, a point of lower and upper monotonicity. Some others, recently published results concerning these points will be shortly indicated.

Bibliography

[1] M. Ciesielski, P. Kolwicz and A. Panfil, *Local monotonicity structure of symmetric spaces with applications,* J. Math. Anal. Appl. 409 (2014), 649-662.

[2] M. Ciesielski, P. Kolwicz and R.Płuciennik, *, Local approach to Kadec-Klee properties in symmetric function spaces,* J. Math. Anal. Appl. 426 (2015), 700-726.

Paweł Kolwicz,
Poznań University of Technology,
Institute of Mathematics,
60-965 Poznań, Poland.
pawel.kolwicz@put.poznan.pl

TOEPLITZ AND HANKEL OPERATORS ACTING BETWEEN DISTINCT HARDY SPACES

KAROL LEŚNIK

Abstract

Classical Toeplitz T_a and Hankel H_a operators on Hardy space H^2 are defined by

$$T_a : f \to P(af) \ \ and f :\to P(aJf)$$

where P is the Riesz projection, J is the flip operator and the function

$$a \in L^\infty$$

is called the symbol of T_a and H_A respectively. Theory of such operators acting on H^p spaces, as well as a number of another kind of spaces is very well developed and still widely investigated. However, in this investigations operators are mainly considered to act from one to the same space. Our goal is to present a background for such operators acting between distinct Hardy spaces, i.e. $T_a : H[X] \to H[Y]$, where X, Y are rearrangement invariant spaces and symbols a are allowed to be unbounded functions from the space of pointwise multipliers $M(X, Y)$ We will present analogues of Brown-Halmos and Nehari theorems for such Toeplitz and Hankel operators, respectively.

Karol Leśnik,
Poznań University of Technology,
Institute of Mathematics,
60-965 Poznań, Poland.
karol.lesnik@put.poznan.pl

MAHLER'A MEASURE OF POLYNOMIALS AND POLYNOMIAL INEQUALITIES VIA RADEMACHER PROCESSES

MIECZYSŁAW MASTYŁO

Abstract

In the first part of the talk we will discuss some recent work with A. Defant on polynomials inequalities between the L^p-norm of complex polynomials on the n-dimensional torus and their Mahler's measure. In the second part we will present joint results with R. Szwedek on abstract variants of the Kahane-Salem-Zygmund inequalities for the expectation of the supremum norm of homogeneous Bernoulli polynomials on the unit ball of a Banach space. We combine ideas from stochastic processes and interpolation theory to control increments of a Rademacher process in an Orlicz space via entropy integrals.

Mieczysław Mastyło,
Adam Mickiewicz University in Poznań,
Faculty of Mathematics and Computer Science,
61-001 Poznań, Poland
mastylo@amu.edu.pl

PROJECTIONS COMUTING WITH OPERATORS

PIOTR NIEMIEC

Abstract

Let $\mathcal{U}_n$ be the set of all subspaces in $\mathbf{R}^n$ and let $\mathcal{B}_n$ be the set of all $n \times n$ real symmetric and idempotent matrices. We construct a bijective map $\Gamma\colon \mathcal{U}_n \to \mathcal{B}_n$ whose restriction to the set $\mathcal{U}_{n,k}$, $k \leq n$, of k-dimensional subspaces in $\mathbf{R}^n$ gives us a bijection onto the set $\mathcal{B}_{n,k}$ of $n \times n$ symmetric matrices with rank k (or trace k). We also give a characterization of the coefficients λ_k^n associated to the projection constant, using the spectral radius of matrices in $\mathcal{B}_{n,k}$. Finally we describe projections onto subspaces of $\mathbf{R}^n$ in terms of lateral inverses, generalizing the fact that an orthogonal projection in $\mathbf{R}^n$ is given by the product of a matrix and its pseudoinverse.

Bibliography

[1] P. Niemiec and P. Wójcik, *Applications of amenable semigroups in operator theory*, in preparation.

Piotr Niemiec,
Jagiellonian University,
Department of Mathematics,
30-348 Kraków, Poland.
Piotr.Niemiec@im.uj.edu.pl

APPLICATIONS OF THE THEORY OF ORLICZ SPACES TO VECTOR MEASURES

MARIAN NOWAK

Abstract

Let $(\Omega, \Sigma, \lambda)$ be a finite complete measure space, (E, ξ) be a sequentially complete lcHs and E'_ξ be its topological dual. Let $ca_\lambda(\Sigma, E)$ stand for the space of all λ-absolutely continuous measures $m : \Sigma \to E$. We show that a uniformly bounded subset M of $ca_\lambda(\Sigma, E)$ is uniformly λ-absolutely continuous if and only if for every equicontinuous subset D of E'_ξ, there exists a submultiplicative Young function φ such that the set $\left\{\frac{d(e' \circ m)}{d\lambda} : m \in \mathcal{M}, e' \in D\right\}$ is relatively weakly compact in the Orlicz space $L^\varphi(\lambda)$. As a consequence, we present a generalized Vitali-Hahn-Saks theorem on the setwise limit of a sequence of λ-absolutely continuous vector measures in terms of Orlicz spaces.

Marian Nowak,
University od Zielona Góra,
Faculty of Mathematics, Computer Science and Econometrics,
65-516 Zielona Góra, Poland
M.Nowak@wmie.uz.zgora.pl

AN UNCONDITIONALLY SATURATED BANACH SPACE WITH THE SCALAR-PLUS-COMPACT PROPERTY

ANNA PELCZAR-BARWACZ

Abstract

A Banach space X has the scalar-plus-compact property if any bounded operator on X is of the form $\lambda Id_X + K$ for some scalar λ and compact operator K. In 2011 S.A. Argyros and R. Haydon constructed the first known space of this kind solving several long-open problems on Banach space structure. The Argyros-Haydon space lacks the regular structure even on subspace level, admitting no infinite unconditional basic sequence. However, the scalar-plus-compact property does not exclude having extremely regular subspaces, even isomorphic to a Hilbert space (S.A. Argyros, D. Freeman, R. Haydon, E. Odell, Th. Raikoftsalis, Th. Schlumprecht, D. Zisimopoulou, 2012).

We will discuss the first example of a Banach space with the scalar-plus-compact property that is saturated by subspaces with regular structure, more precisely every its infinitely dimensional subspace contains an infinite unconditional basic sequence. The space is built within the Argyros-Haydon framework of Bourgain-Delbaen $\mathscr{L}_\infty$-spaces. The talk is based on the joint work with A. Manoussakis and M. Świętek.

Bibliography

[1] A. Manoussakis, A. Pelczar-Barwacz and M. Świętek, *An unconditionally saturated Banach space with the scalar-plus-compact property*, J. Funct. Anal. 272 (12) (2017), 4944–4983

Anna Pelczar-Barwacz,
Jagiellonian University,
Department of Mathematics,
30-348 Kraków, Poland.
Anna.Pelczar-Barwacz@im.uj.edu.pl

SOME RESULTS ON THE EXISTENCE AND MINIMALITY OF SHAPE-PRESERVING EXTENTIONS

MICHAEL PROPHET

Abstract

This talk will describe some aspects of approximation by shape-preserving operators. We will be interested in questions regarding existence and goodness of approximation. Our setting will typically be as follows: let X denote a (real) Banach space, X^* its dual space and V an n-dimensional subspace of X. We want to approximate elements of X with elements from V via a (linear) operator P such that if $f \in X$ has a particular characteristic then so does its approximation Pf. This characteristic might be a shape, expressible using dual approach. Specifically, let $S^* \subset X^*$ be a weak* closed *cone* (a convex set

closed under non-negative scalar). Then we say that $f \in X$ *has shape* if $\langle f, \phi \rangle \geq 0$ for all $\phi \in S^*$. Let $S \subset X$ denote the cone of elements with shape. Thus we are interested in operators $P : X \to V$ which are *shape-preserving* (i.e., $PS \subset S$) and produce good approximations to elements of X.

For fixed X, V, and S, there is immediately the question of existence: which operators $P : X \to V$ leave S invariant? We will give some specific existence results for operators with particular actions on V . In particular, we will examine the existence (and non-existence) of shape-preserving *projections* (operators with the identity action of V).

Concerning the question of goodness of approximation, we confine our discussion to the case in which P is a projection. Here we look to produce good approximations by minimizing $\|P\|$; that is, we look for *minimal shape-preserving projections.* We will discuss several cases in which the existence theory dictates the form of shape-preserving projections and, as such, allows us to give specific formulas for minimal shape-preserving projections.

Michael Prophet,
University of Northern Iowa,
Department of Mathematics,
Cedar Falls, IA 50613, USA,
prophet@math.uni.edu

ON THE MAXIMAL (AND ALMOST MAXIMAL) RELATIVE PROJECTION CONSTANTS

LESŁAW SKRZYPEK

Abstract

We will provide background and survey some earlier results focused on the maximum of relative projection constants over all m-dimensional subspaces of the N-dimensional coordinate space. This quantity, called maximal relative projection constant, translates to the problem of maximizing the sum of the m largest eigenvalues of an N-by-N matrix. Surprisingly (or not) this problem exhibits some interesting connections to other fields like graph theory (Seidel matrices, Taylor graphs and the properties of their adjacency matrix) and equiangular tight frames where answers would depend on Hadamard and Goormaghtigh conjectures. Based on our approach we will demonstrate famous Kadec-Snobar inequality and its refinements. We will also show how to use some random probabilistic techniques (famous semi-circle law) to gain better understanding of the situation and show how to partially reverse the Kadec-Snobar inequality when N does not tend to infinity. Based on joint work with Simon Foucart (Texas A&M)

Lesław Skrzypek
University of South Florida,
Department of Mathematics & Statistics,
Tampa, FL, 33620 USA
skrzypek@usf.edu

COMPLETENESS OF CERTAIN SPACES OF PROBABILITY MEASURES ON $\mathbb{R}^N$ IN A FOURIER-BASED METRICS

MAŁGORZATA STAWISKA-FRIEDLAND

Abstract

In a normed space (X, ν), a best approximation of $x \in X$ in a closed set $C \subset X$ is an $y \in C$ such that $\nu(x - y) = \text{dist}(x, C)$.

In this talk we consider problems related to best approximation in the case when C is a closed semi-algebraic set in the space $X = \mathbb{R}^n$ endowed with a strictly convex semi-algebraic norm ν. In particular, we discuss possible definitions of the critical point correspondence and the distance degree of C.

Małgorzata Stawiska-Friedland
Mathematical Reviews,
Ann Arbor, MI 48103, USA .
stawiska@umich.edu

ON THE PEŁCZYŃSKI CONJECTURE ON AUERBACH BASES

MICHAŁ WOJCIECHOWSKI

Abstract

We consider Auerbach bases in a Banach spaces of dimension $n > 2$. We show that there exist at least $(n - 1)n/2 + 1$ such bases. This estimate follows from the calculation of the Lusternik–Schnirelmann category of the flag variety. A better estimate is obtained for generic smooth Banach spaces using Morse theory.

Michał Wojciechowski,
Institute of Mathematics,
Polish Academy of Sciences,
00-956 Warszawa, Poland
miwoj-impan@o2.pl