

Holomorphic functions on discrete sets in one and several variables

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The study of holomorphic functions on \mathbb{Z}^2 has a history of more than sixty years.

The pioneer was Rufus Philip Isaacs, who introduced two difference equations, both of which are discrete counterparts of the Cauchy–Riemann equation in one complex variable. He thus defined two classes of holomorphic functions on the Gaussian integers $\mathbb{Z}[i] = \mathbb{Z} + i\mathbb{Z}$, called monodiffric functions of the first and second kind, respectively (1941:179). In a later paper (1952) he pursued the study of the monodiffric functions of the first kind.

Jacqueline Ferrand (1944) investigated the monodiffric functions of the second kind, which she called préholomorphes ‘preholomorphic’.

In this lecture I shall

- prove that the Cauchy–Riemann equations have solutions;
- determine the domains of holomorphy in one discrete variable; and
- study the Hartogs phenomenon in two discrete variables.