CONTINUITY PROPERTIES OF FINELY PLURISUBHARMONIC FUNCTIONS

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The fine topology on an open set $\Omega \subset \mathbb{R}^n$ is the coarsest topology that makes all subharmonic functions on Ω continuous. A finely subharmonic function is defined on a fine domain, upper semi-continuous with respect to the fine topology, and satisfies an appropriate modification of the mean value inequality. Fuglede proved the following three properties that firmly connect fine potential theory to classical potential theory: finely subharmonic functions are finely continuous (so there is no super-fine topology), all finely polar sets are in fact ordinary polar sets, and bounded finely subharmonic functions can be uniformly approximated by subharmonic functions on suitable compact fine neighborhoods of any point in their domain of definition. Another continuity result is the *Brelot Property*, i.e. a finely subharmonic function is continuous on a suitable fine neighborhood of any given point in its domain.

In this talk we discuss these matters in the setting of pluripotential theory. The pluri-fine topology on $\Omega \subset \mathbb{C}^n$ is the coarsest topology that makes all plurisubharmonic (PSH) functions on Ω continuous. Finely PSH functions are plurifinely upper semicontinuous functions, the restriction of which to complex lines are finely subharmonic. We will prove the analogs of two of the results mentioned above. Bounded finely PSH functions can locally be written as differences of ordinary PSH functions, hence finely PSH functions are pluri-finely continuous; Next, finely pluripolar sets are pluripolar. As a corollary we obtain that zero sets of finely holomorphic functions of several complex variables are pluripolar sets. We also prove a weak form of the Brelot Property. However the analog of the approximation property remains unknown.

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