Sets in $\mathbb{C}^N$ with vanishing global extremal function

by

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ABSTRACT. Let Γ be a compact non-pluripolar set in $\mathbb{C}^N$. Let f be a function holomorphic in a connected open neighborhood G of Γ. Let (P_n) be a sequence of polynomials with $\deg P_n \le d_n$ $(d_n < d_{n+1})$ such that

$$\limsup_{n\to\infty} |f(z) - P_n(z)|^{1/d_n} < 1, \ z \in \Gamma. \tag{1}$$

We show that if

$$\limsup_{n\to\infty} |P_n(z)|^{1/d_n} \le 1, \ z \in E, \tag{2}$$

where E is a set in $\mathbb{C}^N$ such that the global extremal function $V_E \equiv 0$ in C^N, then the maximal domain of existence G_f of f is one-sheeted, and

$$\limsup_{n\to\infty} \|f - P_n\|_K^{1/d_n} < 1 \tag{3}$$

for every compact set $K \subset G_f$. If, moreover, the sequence (d_{n+1}/d_n) is bounded then $G_f = \mathbb{C}^N$.

If E is a closed set in $\mathbb{C}^N$ then $V_E \equiv 0$ if and only if each series of homogeneous polynomials $\sum_{j=0}^{\infty} Q_j$, for which some subsequence (s_{n_k}) of partial sums converges pointwise on E, possesses Ostrowski gaps relative to a subsequence (n_{k_l}) of the sequence (n_k).

In one-dimensional setting these results are due to J. Müller and A. Yavrian [*On polynomial sequences with restricted growth near infinity*, Bull. London Math. Soc 34(2002), 189-199].