ON A MONGE-AMPÈRE TYPE EQUATION IN THE CEGRELL CLASS $\mathcal{E}_\chi$

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Definition 0.1. Let $\Omega \subseteq \mathbb{C}^n$ be a bounded hyperconvex domain. Define $\mathcal{E}_0$ ($= \mathcal{E}_0(\Omega)$) to be the class of bounded plurisubharmonic functions φ defined on Ω, such that

$$\lim_{z\to\xi} \varphi(z) = 0\,,$$

for every $\xi \in \partial\Omega$, and

$$\int_\Omega (dd^c\varphi)^n < +\infty\,.$$

Define $\mathcal{E}$ ($= \mathcal{E}(\Omega)$) to be the class of plurisubharmonic functions φ defined on Ω, such that for each $z_0 \in \Omega$ there exists a neighborhood ω of z_0 in Ω and a decreasing sequence $[\varphi_j]$, $\varphi_j \in \mathcal{E}_0$, which converges pointwise to φ on ω and

$$\sup_j \int_\Omega (dd^c\varphi_j)^n < +\infty\,.$$

Benelkourchi, Guedj, and Zeriahi introduced the following notation of the Cegrell classes. Let $\chi : (-\infty, 0] \to (-\infty, 0]$ be a continuous and nondecreasing function and let $\mathcal{E}_\chi$ contain those plurisubharmonic functions u for which there exists a decreasing sequence $u_j \in \mathcal{E}_0$, which converges pointwise to u on Ω, as j tends to $+\infty$, and

$$\sup_j \int_\Omega -\chi(u_j)(dd^c u_j)^n < \infty\,.$$

It was proved by Benelkourchi that if $\chi : (-\infty, 0] \to (-\infty, 0]$ is a continuous increasing, convex or concave function such that $\chi(0) = 0$ and $\lim_{t\to-\infty} \chi(t) = -\infty$, then $\mathcal{E}_\chi \subset \mathcal{E}$.

We prove the following theorem.

Main Theorem. *Assume that Ω is a bounded hyperconvex domain in $\mathbb{C}^n$, $n \geq 1$, and let $\chi : (-\infty, 0] \to (-\infty, 0]$ be a continuous increasing function such that $\chi(0) = 0$ and $\lim_{t\to-\infty} \chi(t) = -\infty$, such that $\mathcal{E}_\chi \subset \mathcal{E}$. If μ is a positive and finite measure in Ω, such that $\mu(P) = 0$, for all pluripolar sets $P \subset \Omega$, then there exists a uniquely determined function $u \in \mathcal{E}_\chi$ such that*

$$-\chi(u)(dd^c u)^n = d\mu\,.$$