**Definition 1** (Bergman kernel)**.** Let $A^2(\Omega)$ denotes Bergman space, i.e. $A^2(\Omega) = \mathcal{O}(\Omega) \cap \mathcal{L}^2(\Omega)$. The Bergman kernel $K_\Omega(z, w)$ is the reproducing kernel of $A^2(\Omega)$, given by the Riesz theorem:

$$\forall\ f \in A^2(\Omega),\ \forall z \in \Omega\ f(z) = \int\limits_{\Omega} K_\Omega(z, w) f(w)\ dV.$$

**Definition 2** (Szegö kernel)**.** Let $H^2(\Omega)$ denotes Hardy space. the Szegö kernel $S_\Omega(z, w)$ is the reproducing kernel of $H^2(\Omega)$:

$$\forall\ f \in H^2(\Omega)\ \forall z \in \Omega\ f(z) = \int\limits_{\partial\Omega} S_\Omega(z, w) f(w)\ dS.$$

Main goal of the lecture was to prove the following theorem

**Theorem 1** (see [1])**.** *Let $\Omega \subset\subset \mathbb{C}^n$ be a pseudoconvex domain with $\mathcal{C}^2$-smooth boundary. Then for any $a \in (0, 1)$, there exists a constant $C > 0$ such that*

$$\frac{S(z, z)}{K(z, z)} \leq C\delta(z)|\log(\delta(z))|^{\frac{n}{a}},$$

*where $\delta(z)$ denotes distance to the boundary $\partial\Omega$.*

Proof of this theorem is based on the proof of Błocki's theorem from [2]:

**Theorem 2.** *Let $\Omega$ be a bounded domain in $\mathbb{C}^n$, where we can find $v \in PSH(\Omega)$ and positive constants $A$, $B$, $a$, $b$ such that the following estimate holds in $\Omega$:*

$$\frac{1}{A}\delta^a \leq |v| \leq B\delta^b.$$

*Then there exists positive constants $C_1$, $C_2$, depending only on $n$, $A$, $B$, $a$, $b$ and $R$ - the diameter of $\Omega$, such that for every $\zeta$, $w \in \Omega$ with $\delta(\zeta) \leq e^{-2}$, $\delta(w) \leq e^{-2}$ we have:*

$$|g_\Omega(\zeta, w)| \leq \begin{cases} C_1 \frac{\delta^b(\zeta)}{\delta^a(w)} \log \frac{1}{\delta(w)} & \text{if} \quad \delta(\zeta) \leq \frac{\delta(w)}{2} \\ C_2 \frac{\delta^{\frac{b}{n}}(w)}{\delta^{\frac{a}{n}}(\zeta)} (\log \frac{1}{\delta(w)})^{1-\frac{1}{n}} (\log \frac{1}{\delta(\zeta)})^{\frac{1}{n}} & \text{if} \quad \delta(\zeta) \geq 2\delta(w) \end{cases},$$

*where $g_\Omega$ denotes pluricomplex Green function of $\Omega$.*

## References


[1] Bo-Yong Chen, Siqi Fu, *Comparison of the Bergman and Szegö kernels*, to appear.

[2] Z. Błocki, *The Bergman metric and the pluricomplex Green function*, Trans. AMS 357 (2004), 2613-2625.