AN ESSAY ON BERGMAN COMPLETENESS

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Abstract. We use L^2 estimates to give a effective criterion on Bergman completeness of Stein manifold. Then we give some applications of it, specially, we give a generalization of a famous result of Siu.

Introduction.

Let X be a complex manifold of dimensional n. Following S. Kobayashi [15], we may define the Bergman space $\mathbb{H}(X)$ of holomorphic n-forms f on X satisfying $|\int_X f \wedge \overline{f}| < \infty$. The Bergman kernel form is defined by

$$K_X(x, y) = \sum h_j(x) \wedge \overline{h_j(y)},$$

where $\{h_j\}$ is a complete orthonormal basis of $\mathbb{H}(X)$ (This definition is independent of choice of orthonormal basis). If $K_X(x, x) \neq 0$ for every $x \in X$, then we set

$$ds_X^2 = i \sum \frac{\partial^2 \log K_X^*}{\partial z_\alpha \partial \overline{z}_\beta} dz_\alpha \wedge d\overline{z}_\beta.$$

where $K_X(z, z) = K_X^*(z, z) dz_1 \wedge \cdots \wedge dz_n \wedge d\overline{z}_1 \wedge \cdots \wedge \; d\overline{z}_n$ in local coordinate systems of X. If ds_X^2 is positive, then it is called the Bergman metric of X.

The pluricomplex Green function $g_X(x, y)$ of X with logarithmic pole at y is defined as

$$g_X(x, y) = \sup\{u(x) : u < 0, u \in PSH(X), u(z) \leq \log|z| + O(1) \text{ in a coordinate patch at y}\}.$$

Then we have our main result

Theorem 1. *If a Stein manifold X possesses the Bergman metric, then it is Bergman complete provided the following condition verified*

For any infinite sequence y_k of points in X which has no adherent point in X, there is a subsequence y_{k_j}, a number $a > 0$ and a continuous volume form dV on X such that for any compact subset K of X, one has

$$\int_{K \cap A_X(y_{k_j}, -a)} dV \to 0 \text{ as } j \to \infty.$$

As a application, we have a generalization of a Theorem of Siu, that is every stein subvariety Y in a complex manifold X admits a fundamental family of Bergman complete Stein neighborhoods of Y in X.