CONVEX CHARACTERIZATION OF LINEARLY CONVEX DOMAINS

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The talk is based on the paper [Nik-Tho 2010].

A domain $D \subset \mathbb{C}^n$ is called *linearly convex* if $\mathbb{C}^n \setminus D$ is a union of affine complex hyperplanes. Any set of the form

$$\{z + \lambda X : |\lambda| \leq r\}, \ z \in \mathbb{C}^n, \, X \in (\mathbb{C}^n)_*, \, r > 0$$

is a *disc*. The aim is to prove the following

THEOREM 1. *Let $D \subset \mathbb{C}^n$ be a $\mathcal{C}^{1,1}$-smooth bounded domain and let U be a neighborhood of ∂D. If D contains the convex hull of any two discs in $D \cap U$ with common center then D is linearly convex.*

From the proof of Theorem 1 it follows that a bounded $\mathcal{C}^{1,1}$-smooth bounded domain D is not linearly convex if and only if there are $c \in \partial D$ and a line segment $[a, b]$ in the complex affine tangent hyperplane at c such that

$$c = \frac{a + b}{2} \quad \text{and} \quad [a, b] \setminus \{c\} \subset D.$$

This is analogous to the situation for the real convexity stated in

THEOREM 2. *Let U be a neighborhood of the boundary of a domain $D \subset \mathbb{R}^n$ such that if $D \cap U$ contains two sides of a triangle then it contains the midpoint of the third side. Then D is convex.*

References

[Nik-Tho 2010] N. Nikolov, P. J. Thomas, *"Convex" characterization of linearly convex domains*, arXiv:1009.5022v2 [math.CV].