Definition 1 *A Riemann surface is planar if any smooth closed 1-form with compact support on X is exact.*

We have proved the following theorems:

Theorem 1 (Main Theorem: Koebe 1909) *Any planar Riemann surface is biholomorphic to a domain in the Riemann sphere $\hat{\mathbb{C}}$.*

Theorem 2 *Let X be a planar Riemann surface and $\Omega \subset X$ be an open set in X with compact closure and $\mathcal{C}^\infty$ boundary. Then a closed $\mathcal{C}^\infty$ one-form ω is exact if*

$$\int_{C_i} \omega = 0$$

for any boundary curve C_i.

Theorem 3 (Weyl) *For any path $\gamma_0 : [a, b] \longrightarrow X$ in a Riemann surface X, and any open set U of X containing γ_0, there exists a closed one-form ω_{γ_0} in $X \setminus \{\gamma_0(a), \gamma_0(b)\}$ with support in $U \setminus \{\gamma_0(a), \gamma_0(b)\}$ such that*

(i) $\int_\gamma \omega_{\gamma_0} \in \mathbb{Z}$ *for any closed path γ in $X \setminus \{\gamma_0(a), \gamma_0(b)\}$,*

(ii) *if γ as in* (i) *meets γ_0 in only one point, then*

$$\int_\gamma \omega_{\gamma_0} \in \{-1, 1\},$$

(iii) *if γ as in* (i) *does not meet γ_0, then*

$$\int_\gamma \omega_{\gamma_0} = 0.$$

Theorem 4 *Let X be a non-compact planar Riemann surface, and $\Omega \subset X$ a domain with compact closure and analitic boundary. Then Ω is biholomorphic to a domain in $\mathbb{C}$.*