Let Ω be a bounded domain in $\mathbb{C}^n$.

For $\alpha \,\epsilon\, \mathbb{D},\ a \,\epsilon\, \Omega$ and Ψ be a local indicator. Let τ_Ψ stands for such a nonnegative number for which $(dd^c\Psi)^n = \tau_\Psi \delta_0$ (δ_0 is the Dirac's delta at 0). Then the *multiplicity of* $\varphi \,\epsilon\, \mathcal{O}(\mathbb{D}, \Omega)$ at α with respect to a is given by

$$m_{\varphi,a,\Psi} = \begin{cases} \min\{\tau_\Psi, \liminf_{\zeta\to 0} \frac{\Psi(\varphi(\alpha+\zeta)-a)}{\log|\zeta|}\}, & \text{if } \varphi(\alpha) = a, \\ 0, & \text{otherwise.} \end{cases}$$

Denote by S set of pairs $\{(a_j, \Psi_j) : 1 \leqslant j \leqslant N\}$, where $a_1, \ldots, a_N$ are different points in Ω and $\Psi_1, \ldots, \Psi_N$ are local indicators.

Let $z \,\epsilon\, \Omega$, $\varphi \,\epsilon\, \mathcal{O}(\mathbb{D}, \Omega)$ and $A_j \subset \mathbb{D},\ 1 \leqslant j \leqslant N$. A pair $(\varphi, \{A_j\}_{1\leqslant j\leqslant N})$ is *admissible* (*for* S *and* z) if

$$\varphi(0) = z,\ A_j \subset \varphi^{-1}(a_j) \text{ and } \Sigma_{\alpha \,\epsilon\, A_j} m_{\varphi,a_j,\Psi_j} \leqslant \tau_j,\quad 1 \leqslant j \leqslant N.$$

Then the *generalized Lempert function* is defined by

$$\mathcal{L}_S(z) := \inf\{\Sigma_{j=1}^N \Sigma_{\alpha \,\epsilon\, A_j} m_{\varphi,a_j,\Psi_j}(\alpha) \log|\alpha| \,:\, (\varphi, \{A_j\}_{1\leqslant j\leqslant N}) \text{ is admissible for } S \text{ and } z\}.$$

Theorem Let Ω be a bounded domain in $\mathbb{C}^n$,

$$S := \{(a_j, \Psi_j) : 1 \leqslant j \leqslant N\},\quad S' := \{(a_j, \Psi'_j) : 1 \leqslant j \leqslant N\},\quad \text{where } a_j \,\epsilon\, \Omega,$$

and $\Psi_j,\ \Psi'_j$ are elementary local indicators such that $\Psi_j \leqslant \Psi'_j + C_j$ in a neighborhood of 0, $C_j \,\epsilon\, \mathbb{R},\ 1 \leqslant j \leqslant N$. Then $\mathcal{L}_{S'} \geqslant \mathcal{L}_S$ in Ω.