# SOME CLASSICAL THEOREMS ON GAP POWER SERIES IN MULTIDIMENSIONAL SETTING

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Abstract. Let $f$ be a function of $N$ complex variables holomorphic in a ball $\mathbb{B}(0\,,\,r)$. The function $f$ can be developed either into a series of homogeneous polynomials

$$f(z) \,=\, \sum_{j=0}^{\infty} Q_j(z), \quad z \in \mathbb{B}(0\,,\,r), \tag{1}$$

with

$$Q_j(z) \,:=\, \sum_{|\alpha|=j} \frac{f^{(\alpha)}(0)}{\alpha!} z^{\alpha}, \quad |\alpha| \,:=\, \alpha_1 \,+\ldots+\alpha_N,$$

or into a multiple power series,

$$f(z) \,=\, \sum_{|\alpha|\geq 0} c_\alpha z^\alpha, \quad z \in \mathbb{B}(0\,,\,r), \quad c_\alpha \,:=\, \frac{f^{(\alpha)}(0)}{\alpha!} z^\alpha. \tag{2}$$

Put

$$\psi(z) \,:=\, \limsup_{j\to\infty} \sqrt[j]{|Q_j(z)|}, \quad M(z) \,:=\, \limsup_{j\to\infty} \sqrt[|\alpha(j)|]{|c_{\alpha(j)} z^{\alpha(j)}|},$$

where $\alpha : \mathbb{Z}_+ \ni j \,\to\, \alpha(j) \,\in \mathbb{Z}_+^N$ is a one-to-one mapping.

It is known that $\mathcal{D} := \{z \in \mathbb{C}^N;\, \psi^*(z) \,<\, 1\}$ (resp., $\mathcal{G} \,:=\, \{z \in \mathbb{C}^N;\, M^*(z) \,<\, 1\}$) is a domain of convergence of series (1) (resp., series (2)).

The aim of this talk is to prove the following two theorems.

**Theorem 1.** *If $Q_j \,=\, 0$ for $j \notin \{m_k\}$, where $m_k \,<\, m_{k+1}$, $lim_{k\to\infty} \frac{k}{m_k} \,=\, 0$, then the domain of convergence $\mathcal{D}$ of series (1) is identical with the maximal domain of existence of the function $f$.*

If $N = 1$, Theorem 1 is due to E. Fabry (1896). The proof of Theorem 1, presented in the talk, is a joint result with Professor A. Sadullaev (Tashkent).

**Theorem 2.** *There exists a sequence $\epsilon \,:=\, \{\epsilon_j\}$ with $\epsilon_j \,\in\, \{-1\,,\,1\}$ (resp., a multiple sequence $\epsilon \,:=\, \{\epsilon_\alpha\}$ with $\epsilon_\alpha \,\in\, \{-1\,,\,1\}$ for $\alpha \in \mathbb{Z}_+^N$) such that the function $f_\epsilon \,:=\, \sum_{j=0}^{\infty} \epsilon_j Q_j(z)$ , $z \,\in\, \mathcal{D}$ (resp., the function $f_\epsilon(z) \,:=\, \sum_{|\alpha|\geq 0} \epsilon_\alpha c_\alpha z^\alpha$ , $z \,\in\, \mathcal{G}$) has analytic continuation across no boundary point of the domain of convergence $\mathcal{D}$ of series (1) (resp., domain of convergence $\mathcal{G}$ of series (2)).*

If $N \,=\, 1$, Theorem 2 is due to P. Fatou (1906) and A. Hurwitz - G. Polya (1917).