" Compactness of the $\overline{\partial}$- Neumann operator and applications to Schrödinger and Dirac operators."

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We use a description of precompact subsets in L^2-spaces to characterize compactness of the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is bounded pseudoconvex domain, or

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function.

It turns out that Gårding's inequality in the interior and and the boundary behavior (respetively the behavior at infinity) of suitable plurisubharmonic functions (respectively of the weight function) play a crucial role. Using the general characterization of compactness it is easy to show that property (P) implies compactness of N, if Ω is a smoothly bounded pseudoconvex domain, and that a certain behavior at inifinity of the eigenvalues of the Levi matrix of the weight function φ imply compactness of N_φ.

In the case $L^2(\mathbb{C}^n, e^{-\varphi})$ there is a relationship to Schrödinger operators with magnetic field and Pauli and Dirac operators and to the complex Witten Laplacian. We show that the above results on the $\overline{\partial}$-Neumann operator can be used to settle the question whether these operators have compact resolvent. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C} \text{ entire } : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi} \, d\lambda < \infty\}$$

is infinite-dimensional, which again depends on the behavior at inifinity of the eigenvalues of the Levi matrix of the weight function φ.

F. Haslinger, Compactness for the $\overline{\partial}$- Neumann problem - a functional analysis approach, ESI -preprint 2208, arXiv:0912.4406 , Collectanea Mathematica **62** (2011), 121-129.

F. Haslinger, Compactness of the $\overline{\partial}$- Neumann operator on weighted (0,q)-forms. ESI preprint 2291, arXiv: 1012.433 , Proceedings of the IWOTA Conference 2010, Birkhäuser Verlag, to appear.