# Extension of plurisubharmonic functions with growth control

**Abstract.** The talk is to discuss about main results of the paper with the same name as above written by D.Coman, V.Guedj, A.Zeriahi. Let $X$ be a subvariety of a Stein manifold $M$, a upper semi-continuous function $\varphi : X \longrightarrow [-\infty, +\infty[$ is a plurisubharmonic functions on $X$ if $\varphi \not\equiv -\infty$ and for every $x \in X$ there exists a neighborhood $U$ of $x$ such that $\varphi = u$ on $X$ where $u \in PSH(U)$.

The question is that the morphism: $PSH(M) \longrightarrow PSH(X)$, $\varphi \mapsto \varphi|_X$ is surjective or not? Following Coltoiu's approach, they proved that if $\varphi$ satisfies a growth control then it is always possible, more precisely the following theorem:

**Theorem A.** *Let $X$ be analytic subvariety of a Stein manifold $M$ and let $\varphi$ be a p.s.h function on $X$. Assume that $u$ is a continuous p.s.h exhaution function on $M$ so that $\varphi(z) < u(z)$ for all $z \in X$. Then for every $c > 1$ there exists a p.s.h function $\psi = \psi_c$ on $M$ so that $\psi|_X = \varphi$ and $\psi(z) < c.\max\{u(z), 0\}$ for all $z$ in $M$.*

Note: As explanation in the paper both conditions about growth and $c > 1$ are necessary.
If now we look a similar problem on Kähler manifold with Kähler form $\omega$ for $PSH(V, \omega)$, then class $PSH(X, \omega|_X)$ is also defined, namely, a function semi-continuous $\varphi : X \longrightarrow [-\infty, +\infty[$ is $\omega|_X$-p.s.h if there exists an open covering of $X$ so that $\varphi + \rho_i = \varphi_i \in PSH(U_i)$, where $\rho_i$ is potential of $\omega$ on $U_i$. Then we have the following result:

**Theorem B.** *Let $X$ be a subvariety of a projective manifold $V$ equiped with a Hodge form $\omega$. Then any $\omega|_X-$ p.s.h function on $X$ is the restriction of an $\omega$-p.s.h on $V$.*

From theorem B implies an interesting consequence of approximation of $\omega$-p.s.h on $V$, reads

**Corollary C.** *Let $X$ be a subvariety of a projective manifold $V$ equiped with a Hodge form $\omega$. If $\varphi \in PSH(X, \omega|_X)$ then there exists sequence of smooth functions $\varphi_j \in PSH(V, \omega)$ which decreases pointwise on $V$ so that $\lim \varphi_j = \varphi$ on $X$.*