SEMI-TUBE DOMAINS IN $\mathbb{C}^2$

TOMASZ WARSZAWSKI

A domain $D \subset \mathbb{C}^2$ is called a *semi-tube* if $D = D_0 + i\mathbb{R} \simeq D_0 \times \mathbb{R}$ for some $\mathcal{C}^2$-smooth domain $D_0 \subset \mathbb{R}^3$, which is said to be a *base*.

The following theorems from the paper [1] are proved (see also [2]).

Theorem 1. *Let p be a boundary point of D and suppose that the Levi form of D at p is non-negative* (*resp. positive*) *on the complex tangent space, independently on the position* (*i.e. affine isometry*) *of D_0 in $\mathbb{R}^3$. Then D_0 is convex* (*resp. strongly convex in the analytic sense*) *at p.*

Theorem 2 (Hartogs-Bochner type extension)**.** *Let D_0 be a bounded $\mathcal{C}^{k+1}$-smooth* ($k = 0, 1, 2, \ldots$) *base with connected boundary. Then any $\mathcal{C}^{k+1}$-smooth CR-function on ∂D extends to a function of class $\mathcal{O}(D) \cap \mathcal{C}^k(\overline{D})$.*

Theorem 3. *Let the torus $T = T(R, r)$, where $R > r > 0$, be in a standard position in $\mathbb{R}^3$, that is given by a rotation of a circle $(x_1 - R)^2 + x_3^2 = r^2$ about the x_3-axis. Then a semi-tube $T \times \mathbb{R}$ is*

- *strongly pseudoconvex iff $R/r > \frac{4}{3}\sqrt{\frac{2}{3}}$;*
- *pseudoconvex but not strongly pseudoconvex iff $R/r = \frac{4}{3}\sqrt{\frac{2}{3}}$;*
- *part strongly pseudoconvex and part strongly pseudoconcave iff $R/r < \frac{4}{3}\sqrt{\frac{2}{3}}$.*

References

[1] J. Burgués, R. Dwilewicz, *Geometry of semi-tube domains in* $\mathbb{C}^2$, Adv. Geom. **14** (2012), no. 4, 685–702.

[2] R. Dwilewicz, *Holomorphic extensions in complex fiber bundles*, J. Math. Anal. Appl. **322** (2006), 556–565.