A CHARACTERIZATION OF DOMAINS IN $\mathbb{R}^n\,(\mathbb{C}^n)$ WITH CONICALLY ACCESSIBLE BOUNDARY

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It says usually that a domain $\Omega \subset \mathbb{R}^n$ satisfies the "cone condition" if for every $p \in \Omega$ it includes a closed circular cone $K(p, e(p), \alpha, r)$ with the vertex at the point $p \in \mathbb{R}^n$, an axis vector $e(p)$ and some fixed opening $\alpha\pi, \alpha \in (0,1)$ and height $r \in (0, \infty]$.

The cone condition was used by S. Zaremba in the paper [6] which refers to the Dirichlet problem. Note that in the present year there is the 150 anniversary of birthday of S. Zaremba (1863 - 1942). The cone property and its generalizations are the main tool for solving very important various mathematical problems, for instance see [1].

During the lecture, the author considered domains $\Omega \subset \mathbb{R}^n$ with a property similar to the above cone property. We say that a domain $\Omega \subset \mathbb{R}^n$, including the origin, is $\alpha-$accessible, $\alpha \in [0, 1)$, if for every point $p \in \partial\Omega$ there exists a number $r = r\,(p) > 0$ such that the cone $K(p, p, \alpha, r) > 0$ is included in $\mathbb{R}^n \diagdown \Omega$ ([4]). A fev geometric properties of $\alpha-$accessible domains was given , in particular the following: if $\Omega \subset \mathbb{R}^n$ is an $\alpha-$accessible domain, $\alpha \in (0, 1)$, then for every $p \in \partial\Omega$ and every $\eta \in (0, \alpha)$ there exists a number $r = r\,(p, \eta) > 0$, such that the bounded cone $K(p, -p, \eta, r)$ is included in Ω.

As an application of the above the author demonstrated a solution of the following problem, originated in [3]: characterize all $\alpha-$accessible domains in $\mathbb{C}^N$ which are biholomorphic to the Euclidean ball. Such considerations in $\mathbb{C}^n$ are continuation of some investigations in the complex plane $\mathbb{C}$ (see [5], [2]).

References

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