ON LOCALLY BIHOLOMORPHIC MAPPINGS FROM MULTI-CONNECTED ONTO SIMPLY CONNECTED DOMAINS

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Ligocka [Lig] studied the problem: which domains $D \subset \mathbb{C}$ can be mapped locally biholomorphically onto $\mathbb{C}$ or Δ. She nedeed such mapping from D onto $\mathbb{C}$ to decide if each open Riemann surface X is a Riemann domain over the whole plane $\mathbb{C}$. Ligocka generalized a Gunning-Narasimhan result from [GN] and proved that for every domain $D \subset \mathbb{C}$ there exists a locally biholomorphic mapping from D onto $\mathbb{C}$. Moreover, if D is finitely connected, not biholomorphic to $\mathbb{C} \setminus \{0\}$, then there exists an m-valent, $m \in N$, locally biholomorphic mapping from D onto $\mathbb{C}$. During the talk the author showed that for a class of domains (with an isolated boundary fragment of type I, II orIII), wider than the class of finitely connected domains, there exist a universal bound $m \leq M$ for the $m-$valence of locally biholomorphic mapping from D onto $\mathbb{C}$, and $M = 3$ is the best possible such constant [LS1], [Sta].

The case $f(D) = \Delta$ refers to the following Fornaess-Stout result [FS]: For every paracompact connected $n-$ dimensional complex manifold X there exists a locally biholomorphic mapping from the open unit polydisc Δ^n onto X with the property that every fibre $f^{-1}(x), x \in X$, consists of not more than $(2n+1)4^n + 2$ points. Ligocka [Lig] replaced the polydisc Δ^n in this result by a Cartesian product $D_1 \times ... \times D_n$, of multi-connected domains $D_j, j = 1, ..., n$, but at a cost of worse estimation of the valence: $m \leq (24)^n[(2n+1)4^n + 2]$. This result follows from her theorem that each domain $D \subset \mathbb{C}$, whose complement $\overline{\mathbb{C}} \setminus D$ has an isolated component not a singleton, can be mapped onto Δ locally biholomorphically and $m-$valently, where $m \leq 24$.

During the talk the author showed that for a class of domains with an isolated boundary fragment of the type I or II there exist a universal bound $m \leq M$ for the $m-$valence of locally biholomorphic mapping from D onto Δ, and $M = 3$ (see [LS2], [Sta]). Hence, also the result: If $X = D_1 \times ... \times D_n$, where domains D_j, $j = 1, ..., n$, fulfil the assumptions of the previous result, and Y is a connected paracompact $n-$dimensional complex manifold, then there exists a locally biholomorphic and $m-$valent mapping f from domain X onto manifold Y and $m \leq 3^n[(2n+1)4^n + 2]$).

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