Plurisubharmonic Subextensions As Envelopes Of Disc Functionals

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In this talk we prove a disc formula for the largest plurisubharmonic subextension of an upper semicontinuous function on a domain W in a Stein manifold to a larger domain X under suitable conditions on W and X. We use our disc formula to generalise Kiselman's minimum principle. We show that his infimum function is an example of a plurisubharmonic subextension. Before to state the main theorem let's recall that :
A W-disc structure on X is a family $\beta = (\beta_\nu)$ of continuous maps $\beta_\nu : U_\nu \to \mathcal{A}_X^W$, where (U_ν) is an open cover of X, satisfying the following two conditions. For all $x \in U_\nu, \beta_\nu(0) = x$.
(S) If $x \in U_\nu \cap U_\mu$ then $\beta_\nu(x)$ and $\beta_\mu(x)$ are centre-homotopic. We will be interested in the following condition that β may or may not satisfy.
(N)There is μ such that $U_\mu = W$ and $\beta_\mu(w)$ is the constant disc at w for all $w \in W$.
We say that X is a schlicht disc extension of W if X carries a W-disc structure satisfying (N).

Theorem 1. *Let $W \subset X$ be domains in $\mathbb{C}^n$ such that X is a schlicht disc extension of W. If $\phi : W \to [-\infty, \infty[$ is upper semicontinuous, then*

$$S\phi = E_{\mathcal{A}_X^W}\phi,$$

where $E_{\mathcal{A}_X^W}\phi = inf\left\{ \frac{1}{2\pi}\int_0^{2\pi} \phi o f(e^{i\theta})d\theta, f \in \mathbb{O}(\overline{\mathbb{D}}, X), f(0) = x, f(\mathbb{T}) \subset W \right\}.$

Références

[1] Poletsky, E. A. Plurisubharmonic functions as solutions of variational problems. Several complex variable and complex geometry.

[2] Kiselman, C.O. The partial Legendre transformation for plurisubharmonic function.