RELATIVE EXTREMAL FUNCTIONS 
AND CHARACTERIZATION OF PLURIPOLAR SETS 
IN COMPLEX MANIFOLDS 

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Abstract. We study a disc formula for the relative extremal function for a subset of a complex manifold and apply it to give a description of pluripolar sets and polynomial hulls.

1. Introduction

Let $X$ be a complex manifold and $PSH(X)$ denote the class of all plurisubharmonic functions on $X$, $\mathbb{D}$ the unit disc. We say that $X$ is a Josefson manifold if every pluripolar subset of $X$ is globally pluripolar. The main purpose of this paper is to characterize nonpluripolar sets in the language of analytic discs.

Theorem 1. Let $X$ be a Josefson manifold then:
Every bounded plurisubharmonic function on $X$ is constant, if and only if, for every nonpluripolar subset $A$ of $X$, every point $p \in X$, and every positive $\epsilon$, there exists $f \in \mathcal{O}(\mathbb{D},X) \cap C(\bar{\mathbb{D}},X)$ such that $f(0) = p$ and
$$\lambda(\{\theta \in [0,2\pi); f(e^{i\theta}) \in A\}) > 2\pi - \epsilon,$$
where $\lambda$ denotes the outer Lebesgue measure on $\mathbb{R}$.

Recall that for a bounded pseudoconvex domain $D$ and a compact set $K \subset D$ we have $\hat{K} = \hat{K}_{PSHD}$, where $\hat{K}$ denotes the polynomial hull of $K$. The following result is a basic tool in characterization of polynomial hulls using analytic discs.

Theorem 2. Let $X$ be a complex manifold and let $A$ be a subset of $X$. Then
$$\hat{A}_{PSHX} = \{x \in X: \omega(x,A,X) = -1\}$$
where
$$\hat{A}_{PSHX} = \{x \in X: u(x) \leq \sup_{A} u \text{ for all } u \in PSH(X), u \leq 0\}$$
and
$$\omega(x,A,X) = \sup\{u(x); u \in PSH(X), u \leq -X_{A}\}$$
where $X_{A}$ is the characteristic function of the set $A$.

References