RELATIVE EXTREMAL FUNCTIONS AND CHARACTERIZATON OF PLURIPOLAR SETS IN COMPLEX MANIFOLDS

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ABSTRACT. We study a disc formula for the relative extremal function for a subset of a complex manifold and apply it to give a description of pluripolar sets and polynomial hulls.

1. Introduction

Let X be a complex manifold and $PSH(X)$ denote the class of all plurisubharmonic functions on X, $\mathbb{D}$ the unit disc.
We say that X is a Josefson manifold if every pluripolar subset of X is globally pluripolar. The main purpose of this paper is to characterize nonpluripolar sets in the language of analytic discs.

Theorem 1. *Let X be a Josefson manifold then:*
Every bounded plurisubharmonic function on X is constant, if and only if, for every nonpluripolar subset A of X, every point $p \in X$, and every positive ϵ , there exists $f \in \mathbb{O}(\mathbb{D}, X) \cap C(\overline{\mathbb{D}}, X)$ such that $f(0) = p$ and

$$\lambda(\{\theta \in [0, 2\pi); f(e^{i\theta}) \in A\}) > 2\pi - \epsilon,$$

where λ denotes the outer Lebesgue measure on $\mathbb{R}$.

Recall that for a bounded pseudoconvex domain D and a compact set $K \subset D$ we have $\hat{K} = \hat{K}_{PSHD}$, where $\hat{K}$ denotes the polynomial hull of K. The following result is a basic tool in characterization of polynomial hulls using analytic discs.

Theorem 2. *Let X be a complex manifold and let A be a subset of X. Then*

$$\hat{A}_{PSHX^-} = \{x \in X : \omega(x, A, X) = -1\}$$

where

$$\hat{A}_{PSHX^-} = \{x \in X : u(x) \leq \sup_A u \text{ for all } u \in PSH(X), u \leq 0\}$$

and

$$\omega(x, A, X) = \sup\{u(x); u \in PSH(X), u \leq -X_A\}$$

where X_A is the characteristic function of the set A.

References

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