A DISC FORMULA FOR PLURISUBHARMONIC SUBEXTENTIONS

Abstract. Let $X \subset \mathbb{C}^n$ be a domain and $W \subset X$ a subdomain. Suppose that φ_1 is upper semicontinuous on X and φ_2 is upper semicontinuous on W. Under suitable conditions on W and X, we will give a disc formula for the function

$$E(x) = \sup\{v(x) : v \in PSH(X), v \leq \varphi_1 \text{ on } X, v \leq \varphi_2 \text{ on } W\}.$$

In case $X = W$ and $\varphi_1 = \varphi_2$ we get the classical result of Poletsky. In case when $\varphi_1 = \varphi_2$ on W and φ_1 is big enough on $X \setminus W$ our formula looks as a subextention result of Larusson-Poletsky. In some sense we have a generalization of these results. Our disc formula can also be seen as a generalization of Poletsky's classical result to the situation where the kernel of Poisson functional is not upper semicontinuous on X.

1. Disc formula

Let X be a domain in complex affine space $\mathbb{C}^n$ and $W \subset X$ a subdomain. Consider two upper semicontinuous functions $\varphi_1 : X \longrightarrow \mathbb{R} \cup \{-\infty\}$ and $\varphi_2 : W \longrightarrow \mathbb{R} \cup \{-\infty\}$. We define $\varphi : X \longrightarrow \mathbb{R} \cup \{-\infty\}$ on setting $\varphi = \varphi_1$ on $X \setminus \overline{W}$ and $\varphi = \min\{\varphi_1^*, \varphi_2^*\}$ on $\overline{W}$. Notice that φ is not necessarily upper semicontinuous on X. We take the constant function $-\infty$ to be plurisubharmonic. Let

$$E_1(x) = \sup\{v(x), v \in PSH(X), v \leq \varphi\}.$$

Let $\mathbb{D}$ denote the unit disc, $\mathbb{T}$ the unit circle and σ the arc length measure on $\mathbb{T}$. Set

$$\mathscr{B}_2 = \{f \in \mathscr{O}(\overline{\mathbb{D}}, X), f(\mathbb{T}) \subset X \setminus \overline{W}\}$$

$$\mathscr{B}_1 = \{f \in \mathscr{O}(\overline{\mathbb{D}}, X), f(\mathbb{T}) \subset W\}.$$

Set $\mathscr{B} = \mathscr{B}_1 \cup \mathscr{B}_2$. Suppose that for all $x \in X$ there is $f \in \mathscr{B}$ such that $f(0) = x$.

Theorem 1. *Assume that*

(i) *$\mathscr{B}$ covers X,*

(ii) *For all $x \in \partial W$, $\epsilon > 0$, $r > 0$ small, there is $f \in \mathscr{O}(\overline{\mathbb{D}}, B(x, r))$ such that*

$$\sigma(\{t \in \mathbb{T}, f(t) \in B(x, r) \cap W\}) > 1 - \epsilon.$$

Then $E_1(x) = E(x) = \inf\left\{\frac{1}{2\pi}\int_0^{2\pi} \varphi \circ f(e^{i\theta})d\theta, f \in \mathscr{O}(\overline{\mathbb{D}}, X), f(0) = x\right\}$.

I will also present some basic properties of the function $\omega(., A, D)$, see[3,4], where,

$$\omega(z, A, D) = \sup\{u(z) : u \in PSH(D) : u \leq 0 \text{ on } D, \limsup_{w \to \zeta} u(w) \leq -1 \text{ for any } \zeta \in A\}.$$

References

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