ON ASYMPTOTIC EXPANSIONS FOR LINE BUNDLES ON COMPLEX MANIFOLDS

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Abstract. The talk concerns tensor power series $L^{\otimes N}$ of line bundles $L \to \mathcal{M}$ over a closed compact or projective algebraic Kähler manifold $\mathcal{M}$. We focus on the results of Steve Zelditch [1] and Gang Tian [2] on diverse asymptotics for $L^{\otimes N}$ as $N \to \infty$. The theorem of Tian relates the Hermitian metric $h_N = h^N$ on $L^{\otimes N}$ induced by a metric h on $L \to \mathcal{M}$ and the Fubini-Study metric h_{FS} inherited from the projective structure of the manifold $\mathcal{M}$. Namely, the asymptotic formula

$$\max_{\mathcal{M}} \left\{ \left\| h_N - \frac{1}{N} h_{FS} \right\|_{\mathcal{M}}, \ \left\| D\Big(h_N - \frac{1}{N} h_{FS} \Big) \right\|_{\mathcal{M}}, \ \left\| D^2\Big(h_N - \frac{1}{N} h_{FS} \Big) \right\|_{\mathcal{M}}, \ \left\| R\Big(h_N - h_{FS} \Big) \right\|_{\mathcal{M}} \right\} = O\left(\frac{1}{\sqrt{N}} \right),$$

where D is the covariant derivative and R is the curvature tensor, holds true. This relation also can be considered as a corollary of the Zelditch theorem, where the corresponding asymptotic formula is derived for an arbitrary orthogonal basis $S_1^N(x)$, $S_2^N(x)$, $\ldots$, $S_{d_N}^N(x)$ in the space of all globally holomorphic sections $H^0(\mathcal{M}, L^{\otimes N})$, $d_N = \dim H^0$, for a closed compact Kähler manifold $\mathcal{M}$. The proof proposed by Zelditch involves diverse fundamental facts and concepts of complex analysis of several variables like pseudoconvexity for dual bundles $L^* \to \mathcal{M}$, analysis on Cauchy-Riemannian manifolds, Hardy spaces and Szegő kernels. The asymptotic expansion is obtained by applying the method of stationary phase to the parametrix for Szegő kernels and based on the results of Boutet de Monvel and Sjöstrand. The formulas have many corollaries and a potential for further studies on complex manifolds.

References

[1] *S. Zelditch*, "Szegő kernels and a theorem of Tian", Int. Math. Res. Not. **6** (1998), pp. 317–331.

[2] *G. Tian*, "On a set of polarized Kähler metrics on algebraic manifolds", J. Differential Geometry **32** (1990), pp. 99–130.

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