

**A GLEASON-KAHANE-ZELAZKO THEOREM FOR  
MODULES AND APPLICATIONS TO HOLOMORPHIC  
FUNCTION SPACES**

THOMAS RANSFORD

We generalize the Gleason-Kahane-Zelazko theorem to modules. As an application, we show that every linear functional on a Hardy space that is non-zero on outer functions is a multiple of a point evaluation. A further consequence is that every linear endomorphism of a Hardy space that maps outer functions to nowhere-zero functions is a weighted composition operator. In neither case is continuity assumed. We also consider some extensions to other function spaces. (Joint work with Javad Mashreghi.)