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Zeros of univariate functions of special form[1]

During this talk we are going to consider the zeros of univariate entire functions of the form $f(z^k) + z^p g(z^k)$ and $g(z^k) + z^p f(z^k)$, where $f(z)$ and $g(-z)$ have genus 0 and only negative zero points. It will be shown that the zeros are simple (or at most double in a special case) and distributed "uniformly" among $2k$ sectors of the complex plane. The approach rests on a connection to the Pick class of functions mapping the upper half of the complex plane into itself (also known as $\mathcal{R}$- or Nevanlinna functions).

As an application, we will deduce that functions of the form

$$\sum_{n=0}^{\infty} (\pm i)^{\frac{n(n-1)}{2}} a_n z^n, \quad \text{where} \quad a_n \geq 0,$$

under a certain condition on the coefficients have simple zeros distinct in absolute value. This includes the "Disturbed Exponential" function corresponding to

$$a_n = \frac{1}{n!} \cdot q^{\frac{n(n-1)}{2}} \quad \text{when} \quad 0 < q \leq 1,$$

as well as the Partial Theta function corresponding to

$$a_n = q^{\frac{n(n-1)}{2}} \quad \text{when} \quad 0 < q \leq q_* \approx 0.7457224107;$$

both appear in problems of statistics and combinatorics (see e.g. the papers by Alan Sokal: *"Some wonderful conjectures..."* of 2009 and *"The leading root of the partial theta function"* of 2012).

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