Optimal regularity for the convex envelope (based on a paper by G. de Philippis and A. Figalli)

The following theorem due to G. de Philippis and A. Figalli was presented:

Let Ω be a uniformly convex domain in $\mathbb{R}^n$ with $\mathcal{C}^{3,1}$-smooth boundary and $f \in \mathcal{C}^{3,1}(\bar{\Omega})$ be given. Then the convex envelope defined by

$$\Gamma_f(x) := sup\{l(x)|\ l-\text{affine},\ l \leq f\}$$

belongs to $\mathcal{C}^{1,1}(\bar{\Omega})$.

Several examples were shown proving that

(1) *The regularity assumptions on f and $\partial\Omega$ are optimal;*
(2) *In general one cannot expect better regularity for the envelope, no matter how smooth the data is.*