It is known that both Carathéodory and Kobayashi pseudodistances depend continuously on increasing sequences of domains (in the latter case, adding some regularity assumptions on limiting domain; cf. [1] and references therein). The pseudodistances mentioned above are particular examples of wider class of holomorphically contractible systems, i.e. systems of functions

$$\boldsymbol{d}_D : D \times D \to [0, +\infty),$$

D running through all domains in all $\mathbb{C}^n$'s, such that $\boldsymbol{d}_{\mathbb{D}}$ is forced to be $\boldsymbol{p}$, the hyperbolic distance on $\mathbb{D}$, the unit disc on the plane and all holomorphic mappings are contractions with respect to the system $(\boldsymbol{d}_D)$. The question about the behaviour of holomorphically contractible systems under not necessarily monotonic sequences of sets seems to be natural and important. We shall present some result stating the continuity of holomorphically contractible systems under the sequences of domains convergent with respect to Hausdorff distance (for two nonempty bounded sets A, B it is defined as

$$\mathfrak{H}(A, B) := \inf\{\delta > 0 : A \subset B^{(\delta)} \text{ and } B \subset A^{(\delta)}\},$$

where for a set S and a positive number ε, the set $S^{(\varepsilon)} := \bigcup_{s \in S} \mathbb{B}(s, \varepsilon)$ is the *ε-envelope of S*; $\mathbb{B}(x, r)$ denotes the open Euclidean ball of center x and radius r). The result under consideration reads as follows:

Theorem ([2]). *Let $(\boldsymbol{d}_D)$ be a holomorphically contractible system and let $D \subset \mathbb{C}^m$ be a bounded domain. Assume that there exist two sequences $(I_n)_{n\in\mathbb{N}}, (E_n)_{n\in\mathbb{N}}$ of domains such that*

$$E_{n+1} \subset\subset E_n, n \in \mathbb{N}, \bigcap_{n\in\mathbb{N}} E_n = \overline{D}, I_n \subset\subset I_{n+1}, n \in \mathbb{N}, \bigcup_{n\in\mathbb{N}} I_n = D$$

and such that for each $z, w \in D$ there is

$$\lim_{n\to\infty} \boldsymbol{d}_{E_n}(z, w) = \lim_{n\to\infty} \boldsymbol{d}_{I_n}(z, w) = \boldsymbol{d}_D(z, w).$$

Let $(D_n)_{n\in\mathbb{N}}$ be a sequence of bounded domains in $\mathbb{C}^m$ such that

$$\lim_{n\to\infty} \mathfrak{H}(\overline{D}_n, \overline{D}) = 0$$

and with the property that for each compact $K \subset D$ there exists an $n_0 \in \mathbb{N}$ such that for any $n \geq n_0, K \subset D_n$. Then for any $z, w \in D$

$$\lim_{n\to\infty} \boldsymbol{d}_{D_n}(z, w) = \boldsymbol{d}_D(z, w).$$

As a supplement, we shall present one of the main results of particular interest from [1], e.g.

Theorem. *Let $D \subset \mathbb{C}^m$ be a bounded domain, each boundary point of which admits a weak peak function. Let $(D_n)_{n\in\mathbb{N}}$ be a sequence of domains in $\mathbb{C}^m$ such that*

$$D_{n+1} \subset\subset D_n, n \in \mathbb{N}, \bigcap_{n\in\mathbb{N}} D_n = \overline{D}.$$

Then for every pair of points $z, w \in D$ we have

$$\lim_{n\to\infty} \boldsymbol{k}_{D_n}(z, w) = \boldsymbol{k}_D(z, w),$$

$\boldsymbol{k}$ standing for the Kobayashi pseudodistance.

References

[1] M. Kobayashi, *Convergence of Invariant Distances on decreasing domains*, Complex Variables 47 (2002), 155-165.

[2] A. Lewandowski, *On bahaviour of holomorphically contractible systems under non-monotonic sequences of sets*, Archiv der Mathematik, to appear.