AUTOMORPHISMS OF A GENERALIZED HARTOGS TRIANGLE

(BASED ON A. KODAMA'S PAPERS [Kod16b] AND [Kod16a])

For $\ell_i, m_j \in \mathbb{N}$ and $p_i, q_j > 0$ with $1 \le i \le I$, $1 \le j \le J$ set

$$\ell = (\ell_1, \dots, \ell_I),\ m = (m_1, \dots, m_J), \quad p = (p_1, \dots, p_I),\ q = (q_1 \dots, q_J)$$

and define *generalized Hartogs triangle*

$$\mathcal{H} = \mathcal{H}^{p,q}_{\ell,m} := \left\{ (z, w) \in \mathbb{C}^N : \sum_{i=1}^{I} \|z_i\|^{2p_i} < \sum_{j=1}^{J} \|w_j\|^{2q_j} < 1 \right\},$$

where

$$\begin{aligned} z &= (z_1, \dots, z_I) \in \mathbb{C}^{\ell_1} \times \dots \times \mathbb{C}^{\ell_I} = \mathbb{C}^{|\ell|},\ |\ell| = \ell_1 + \dots + \ell_I, \\ w &= (w_1, \dots, w_J) \in \mathbb{C}^{m_1} \times \dots \times \mathbb{C}^{m_J} = \mathbb{C}^{|m|},\ |m| = m_1 + \dots + m_J, \\ \mathbb{C}^N &= \mathbb{C}^{|\ell|} \times \mathbb{C}^{|m|},\ N = |\ell| + |m|. \end{aligned}$$

We always assume that $p_2, \dots, p_I \neq 1$ and $q_2, \dots, q_J \neq 1$, if $I \ge 2$ or $J \ge 2$.

Theorem 1. *Let $|m| = 1$. Then $\mathrm{Aut}(\mathcal{H}^{p,q}_{\ell,m})$ consists of all transformations*

$$\Phi : (z_1, \dots, z_I, w) \longmapsto (\tilde{z}_1, \dots, \tilde{z}_I, \tilde{w}),$$

of the following form

Case I. $I = 1$

(I.1) $q/p \in \mathbb{N}$. Then

$$\tilde{z}_1 = w^{q/p} H(z_1/w^{q/p}), \quad \tilde{w} = Bw,$$

where $H \in \mathrm{Aut}(\mathbb{B}_{\ell_1})$ and $B \in \mathbb{T}$.

(I.2) $q/p \notin \mathbb{N}$. Then

$$\tilde{z}_1 = Az_1, \quad \tilde{w} = Bw,$$

where $A \in \mathbb{U}(\ell_1)$ and $B \in \mathbb{T}$.

Case II. $I \ge 2$.

(II.1) $p_1 = 1$, $q \in \mathbb{N}$. Then

$$\tilde{z}_1 = w^q H(z_1/w^q), \quad \tilde{z}_i = \gamma_i(z_1/w^q) A_i z_{\sigma(i)},\ (2 \le i \le I), \quad \tilde{w} = Bw,$$

where $H \in \mathrm{Aut}(\mathbb{B}_{\ell_1})$, γ_i are nowhere vanishing functions on $\mathbb{B}_{\ell_1}$ defined by

$$\gamma_i(z_1) = \left(\frac{1 - \|a\|^2}{(1 - \langle z_1, a \rangle)^2} \right)^{1/(2p_i)}, \quad a = H^{-1}(0),$$

$A_i \in \mathbb{U}(\ell_i)$, $B \in \mathbb{T}$, and σ is a permutation of $\{2, \dots, I\}$ such that $\sigma(i) = s$ iff $(\ell_i, p_i) = (\ell_s, p_s)$.

(II.2) $p_1 \neq 1$ or $q \notin \mathbb{N}$. Then

$$\tilde{z}_i = A_i z_{\sigma(i)},\ (1 \le i \le I), \quad \tilde{w} = Bw,$$

where $A_i \in \mathbb{U}(\ell_i)$, $B \in \mathbb{T}$, and σ is a permutation of $\{1, \dots, I\}$ such that $\sigma(i) = s$ iff $(\ell_i, p_i) = (\ell_s, p_s)$.

Theorem 2. *Let $|m| \ge 2$. Then $\mathrm{Aut}(\mathcal{H}^{p,q}_{\ell,m})$ consists of all transformations*

$$\Phi : (z_1, \dots, z_I, w_1, \dots, w_J) \longmapsto (\tilde{z}_1, \dots, \tilde{z}_I, \tilde{w}_1, \dots \tilde{w}_J),$$

of the form

$$\tilde{z}_i = A_i z_{\sigma(i)},\ (1 \le i \le I), \quad \tilde{w} = B_j w_{\tau(j)},\ (1 \le j \le J),$$

where $A_i \in \mathbb{U}(\ell_i)$, $B_j \in \mathbb{U}(m_j)$, and σ, τ are permutations of $\{1, \dots, I\}$, $\{1, \dots, J\}$, respectively, such that $\sigma(i) = s$ iff $(\ell_i, p_i) = (\ell_s, p_s)$ and $\tau(j) = t$ iff $(m_j, q_j) = (m_t, q_t)$.

References

[Kod16a] A. Kodama, *Correction: On the holomorphic automorphism group of a generalized Hartogs triangle*, Tohoku Math. J. (2) **68** (2016), no. 1, 47–48.

[Kod16b] ———, *On the holomorphic automorphism group of a generalized Hartogs triangle*, Tohoku Math. J. (2) **68** (2016), no. 1, 29–45.